DAMPING OF RESONANCES IN RFI FILTERS
WITH RC MEMBERS

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The resonances in the pass-band of the RFI filter can be suppressed by introducing an RC member in the parallel, capacitive branch of the filter. The paper describes an analytical way of calculating components of the elemental "L" section filter as it works under odd interface conditions 0/0. More complex, multiple "L" section filters can be designed using a digital computer, such that the paper describes principles of the computer program as well. At the end, theoretical results are verified experimentally and the results of measuring are also given.

Introduction

The best method for suppressing conducted RF interference is filtering. The difference between a conventional filter and a filter for suppressing RFI (especially a power-line filter) is that the first one is mostly interface matched, and the second one is often interface mismatched. Resonances and insertion gain (negative value of insertion loss) are the effects of mismatching [1],[2] and [3].

There are several methods for eliminating the resonance problem in RFI filters. In general, resonances are suppressed by introducing frequency-selective losses into the generator, the load, the power line or the filter [1]. In most cases when we design RFI filters, we don't know where the filter will be installed. For that reason, it is much more interesting to introduce losses into the power line or into the filter, than into the generator or into the load in every angular case.

Lossy lines are useful only at very high frequencies and for very long lines [1],[4] and [5].

If losses are introduced into the filter, they may be introduced into a series-inductive or shunt-capacitive branch. The lossy series-inductive branch can be realized by dispersion in the magnetic material of the inductor [4]; by wire losses in the form of increased skin-effect [6] or by adding an RL combination parallel to the inductor. Schlick [1] showed that the first two mechanisms are not useful for frequencies below 5 MHz. The third method is impractical because of enhanced volume and the cost of the filter. Introducing losses into the shunt-capacitive branch can be done by adding an RC member parallel to the capacitor. This paper describes that last method of damping resonances.

Background

Paert [7] described a method of designing filters for interference suppression in phase-controlled regulators. Jamped with an RC member using design curves, but the load must be resistive and we must know the value of it.

Power feed line filters for different purposes are designed according to Schlick's extreme-value theory of mismatch [3]. The main resonance problem for phase-controlled regulators is ringing, and the worst case of oscillating in power feed line filters are eigen(self)-resonances.

If we have odd interface conditions (and in the asymptotic cases the source of interference is an ideal voltage source and the filter is in the open-circuit condition, or vice versa: the source of interference is an ideal current source and the filter is in the short circuit condition) the eigen-resonance problem occurs and we have insertion gain. Practically the insertion gain is high enough if we have instead of 0/0 condition \( Z_o/Z_r < 0.1 \) and \( Z_o/Z_r > 10 \), and instead of 0/0 condition \( Z_o/Z_r < 0.1 \) and \( Z_o/Z_r > 10 \) (\( Z_o \) - impedance of the source of interference, \( Z_r \) - impedance of the load and \( Z_o \) - characteristic impedance of the filter). If we have odd interface conditions, the method of designing filters with an RC member is quite different, and will be described below.
**General considerations**

Characteristics of filters in general can be described with chain parameters.

\[
\begin{bmatrix}
U_1 \\
I_1
\end{bmatrix} = \begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
U_2 \\
I_2
\end{bmatrix}
\]  \(1\)

The insertion loss is:

\[
\text{IL} = 20 \cdot \log |x|  \tag{2}
\]

where \( |x| \) is the insertion voltage ratio and can be expressed in terms of the chain parameters.

\[
|x| = \frac{a_{11} Z_L + a_{12} + a_{21} Z_0 + a_{22} Z_L}{Z_0 + Z_L}  \tag{3}
\]

For odd asymptotic interface conditions the insertion voltage ratio is

\[
Z_0 = 0, \ Z_L \rightarrow \infty \quad |x| = \left| a_{11} \right|  \tag{4}
\]

\[
Z_L = 0, \ Z_0 \rightarrow \infty \quad |x| = \left| a_{22} \right|  \tag{5}
\]

It is sufficient in consideration of the insertion gain problem to analyze just this asymptotic case. Insertion gain is less in real conditions than in the asymptotic conditions, because in real conditions we have additional damping with generator and load resistance.

**The elemental LC filter**

First of all we will analyze the elemental low-pass "L" section filter (LC filter) damped with an RC member. Fig.1 shows this filter.

![Diagram of the elemental LC filter with an RC member](image)

Fig.1: The elemental LC filter damped with an RC member

We will analyze just one odd asymptotic interface condition \( Z_0 = 0, \ Z_L \rightarrow \infty \) because for the other insertion loss is for all frequencies equal to zero \( a_{22} = 1 \).

The chain parameter \( a_{11} \) of the LC filter from figure 1 can be calculated from the equation (1)

\[
a_{11} = \frac{U_1}{U_2} \bigg| _{I_2 = 0} = \frac{X_{C1} X_{C2} + X_{C1} X_L + X_{C2} X_L}{X_{C1} + X_{C2}}  \tag{6}
\]

where

\[
X_L = j \omega L
\]

\[
X_{C1} = \frac{1}{j \omega C_1}
\]

\[
X_{C2} = R + \frac{1}{j \omega C_2}
\]

\[j = \sqrt{-1}, \ \omega = 2 \pi f \text{-radian frequency}\]

The insertion gain is not difficult to show that the absolute value of \( a_{11} \), or insertion voltage ratio \( |x| \) is:

\[
|x| = \sqrt{\frac{A^2 + (B - C)^2}{A^2 + C^2}}  \tag{7}
\]

where

\[
A = \frac{R \varepsilon^2}{(1+\varepsilon)^2 + (\omega R C_1 \varepsilon)^2}
\]

\[
B = \omega L
\]

\[
C = \frac{(1+\varepsilon)/C_1 + C_1 (\omega R \varepsilon)^2}{\omega[(1+\varepsilon)^2 + (\omega R C_1 \varepsilon)^2]}
\]

\[
\varepsilon = C_2 / C_1
\]

\(|x|\) is a function of five parameters \( L, C_1, \omega, \varepsilon \) and \( R \). In one frequency range \(|x|\) will be less than 1, and because of (2) the insertion loss will be negative. We will have insertion gain. In this moment let us suppose that we know \( L, C_1, \varepsilon \) (latter it will be shown how we calculate these components). The problem is to find the value of the resistor \( R \) for which the insertion gain will be minimal. Let's introduce a new variable \( K \) as a quotient of \( B \) and \( C \) from equation (7) \( K = B/C \). Insertion voltage ratio as a function of \( K \) is shown in figure 2.

![Insertion voltage ratio as a function of K](image)

Fig.2: Insertion voltage ratio as a function of \( K = B/C \)

The insertion gain is maximal when \(|x|\) is minimal. From the curve in figure 2 we can see that \(|x|\) is minimal for \( K = 1 \), or \( B = 0 \).
For $K = 1$ equation (7) is:

$$|x_1|_0 = \frac{R\epsilon^2}{\sqrt{(R\epsilon)^2 + [(1 + \epsilon)/\omega C_1 + \omega C_1(\omega \epsilon)]}}$$

(8)

Optimal value of the resistor

We want to find the value of $R$ ($R_0$) for which $|x_1|_0$ from equation (8) will have the maximum (and then the insertion gain will be minimal).

$$\frac{\partial |x_1|_0}{\partial R} = 0$$

(9)

After derivation and simple calculation we obtain the following expression for optimal value of the resistor $R$ ($R_{opt}$):

$$R_{opt} = \frac{1}{\omega_0 C_1 \epsilon} \cdot \sqrt{1 + \epsilon}$$

(10)

$c$ and $\epsilon$ are known, but we still don't know the value of $\omega$ for $K = 1$ and $R = R_{opt}$ ($\omega_0$). The qualitative curve of $K$ as a function of the radian frequency $\omega$ is given in figure 3.

$$K(\omega) = f(\omega) \cdot g(\omega)$$

$$f(\omega) = a \omega^2 + b$$

$$g(\omega) = \frac{-b \omega d}{d + c \omega^2}$$

$$a = L C_1$$

$$b = \frac{1 + \epsilon}{R^2 C_1 \epsilon}$$

$$c = C_1 (\omega \epsilon)$$

$$d = C_1 (1 + \epsilon)$$

Fig. 3: $K$ as a function of the radian frequency $\omega$

From figure 3 it can be seen that the dependence of $K$ and $\omega$ is unique but not linear. If $K = 1$ it follows that $B = C$. From this relationship we can find the expression for $R$ if $K = 1$.

$$R = \frac{1}{\omega C_1 \epsilon} \cdot \frac{1 - \omega^2 L C_1 (1 + \epsilon)}{\omega^2 L C_1 - 1}$$

(11)

For different values of $R$ although $K = 1$, we have different values of $\omega$. If $R = R_{opt}$, equations (10) and (11) must be equal. That means that

$$(1 - \omega^2 L C_1 (1 + \epsilon))/(-\omega^2 L C_1 - 1)$$

from equation (11) must be 1. Now it is easy to find the expression for $\omega$ if $K = 1$, and $R = R_{opt}$.

$$\omega_0 = \frac{1}{\sqrt{L C_1 (1 + \epsilon) / 2}}$$

(12)

From equations (10) and (12) we can write the final equation for the optimal value of the resistor $R$.

$$R_{opt} = \frac{1}{\epsilon} \sqrt{L (1 + \epsilon)(1 + \epsilon) / 2}$$

(13)

The minimal insertion gain

Minimal value of the insertion gain $IG_{min}$ can be calculated from equations (2), (8), (12) and (13) using the knowledge that the insertion gain is the negative value of the insertion loss.

$$IG_{min} = 20 \cdot \log \frac{\epsilon + 2}{\epsilon}$$

(14)

The variation of the insertion voltage ratio $|x_1|_0$ and the minimal insertion gain $IG_{min}$ as a function of $\epsilon$ is shown in figure 4.

![Graph of |x1|_0 and IGmin as functions of epsilon](image)

Fig. 4: $|x_1|_0$ and $IG_{min}$ as functions of $\epsilon$

Now we can plot the dependence of the insertion voltage ratio $|x_1|_0$ and the insertion loss $IL$ in the passband of the filter as functions of the radian frequency $\omega$. $R$ is the parameter (figure 5).

For all cases the insertion loss is negative in one frequency range. The negative insertion loss (insertion gain) is at a minimum when $R = R_{opt}$. For frequencies where $IL > 0$ and for extreme values of $R$ ($R = 0$ and $R = \infty$) the slope of the insertion loss curve is 40 dB per decade. For $0 < R < \infty$ the slope is in the first part less than 40 dB per decade. In the second part the slope is also 40 dB per decade.
FIG. 5: \( |x| \) and \( L \) as functions of \( \omega \) and \( R \)

1. \( R = 0 \)
2. \( 0 < R < R_{\text{OPT}} \)
3. \( R = R_{\text{OPT}} \)
4. \( R_{\text{OPT}} < R < \infty \)
5. \( R \to \infty \)

Calculation of \( L \) and \( C_i \):
At the beginning of this section we supposed that we knew the values of \( L, C_i \) and \( f \). Now it will be shown how we calculate these components.
The maximal voltage drop on the series branch of the filter \( (U_{\text{MAX}}) \) often is given for power feed line filters. This value in most cases determines the value of series inductance \( L \)

\[
L = \frac{U_{\text{MAX}}}{\omega_0 I_n} \tag{15}
\]

\( U_{\text{MAX}} \) - max. voltage drop
\( I_n \) - rated line current
\( \omega_0 = 2 \pi f_n \) - rated radian frequency

The upper limit value of the insertion gain determines the parameter \( \xi \) (Fig. 4). From Fig. 4 we can see that for \( \xi > 10 \) the insertion gain is not much less than for \( \xi = 10 \).

A request for insertion loss in the standband of the filter determines the value of capacitor \( C_i \). For frequencies, high in the standband the influence of \( C_i \) and \( R \) to the insertion loss of the filter can be neglected. If \( Z_0/Z_0 < 0.1 \) and \( Z_2/Z_0 > 10 \) the value of \( C_i \) can be calculated from the simple formula:

\[
C_i = \frac{8}{10^5} \omega^2 L \tag{16}
\]

When we know the value of \( \xi \) and \( C_i \), it is easy to determine the value of \( C_i \), because \( \xi = C_i/C_i \).

All of these results can be also applied to the "\( \Pi \)" section filter (CLC filter) with the RC members in parallel with both capacitors and to the "\( T \)" section filter (LCL filter) with one RC member in parallel with the capacitor under both odd asymptotic interface conditions \( (Z_0 = 0, Z_i \to 0, Z_i \to 0) \).

This method of determining the optimal value of the resistor in the RL member if resonances of the filter are damped with the RL combination in parallel with the inductor instead of the RC combination in parallel with the capacitor. The principle is the same, but the equations are different.

### The multiple LC filter

The analytical analysis of multiple complex filters (multiple LC filters) is impractical because equations are big and nonperceptible upon the first glance. For example if we have a filter with two identical "\( \Pi \)" sections (LCLC filter whose partition number is 2) damped with equal RC members the insertion voltage ratio will be:

\[
|x| = \left[ \frac{X_0 X_1 X_2 + X_1 X_2 X_3 + X_1 X_2 X_3 (X_2 X_3)}{X_1 X_2 X_3} \right] \tag{17}
\]

\( X_0, X_1 \), and \( X_2 \) are given below the equation (6).

### Computer program analysis

It is much more practical to compute the optimal value of the resistor \( R \) on a digital computer using the method of iteration. The optimal value of the resistor \( R_{\text{OPT}} \) increases with the number of "\( \Pi \)" sections (partition number of the filter). So, we begin iteration with \( R \) computed from the formula (13) and increase the value of the resistor in discrete steps. For each value of \( R \) we compute the insertion loss curve and find the maximal value of the insertion gain \( (I_{\text{MAX}}) \). \( R_i \) for which \( I_{\text{MAX}} \) is the least is \( R_{\text{OPT}} \) and that insertion gain is the minimal which can be achieved with the values of \( L, C_i \) and \( C_i \).

### Computer print-outs

Print outs of the computer program for two examples are given in Fig. 6. In both examples we have the same values of inductor \( L \) and capacitors \( C_i \) and \( C_i \). The partition number in the first example is 2 and in the second 4. The first point of the insertion loss curve was calculated at the frequency 1 kHz, and the last at the frequency 100 kHz. In this frequency range the insertion loss was calculated at fifty discrete frequencies.
ELEMENT VALUES OF A FILTER

PARTITION NUMBER=N
SERIES INDUCTOR-L
SHUNT CAPACITOR-C1
RC CAPACITOR-C2
FIRST FREQUENCY-FH
LAST FREQUENCY-FL
OPTIMAL RESISTOR-ROPT
MIN INSERTION GAIN-INGN
FREQUENCY OF IGMIN-FG

1.0±0.3 ohm
100±0.5 F
200±0.5 F
1.2±0.5 Hz
1.0±0.5 Hz
12.5±0.05 UHPA
1.473±0.08
59±0.04 Hz

Element values of a filter

Fig. 7: Schematic diagram of the insertion loss measuring system
1) 50Ω signal generator
2) wide band transformer 22:1
3) filter under test
4) high input impedance RF voltmeter
5) Faraday cage

Results

Insertion loss curves for all four different values of R are shown in figure 8. The insertion loss was measured in the frequency range from 0.5 kHz to 100 kHz.

As we predicted, the minimal insertion gain was for R = Ropt = 120Ω (13). The measured minimal insertion gain was 1.58 dB at the frequency 4.58 kHz. From equations (14) and (12) we can calculate the minimal insertion gain and the frequency where it is located. Results of calculation are: \( R_{\text{min}} = 1.42 \) dB and \( f_{\text{opt}} = \frac{45.08}{2\pi} \) kHz.

For \( R > R_{\text{opt}} \) and \( R < R_{\text{opt}} \), the minimal insertion gain is greater than for \( R = R_{\text{opt}} \) and it is at a maximum for \( R = 0 \) and \( R \to \infty \). Theoretically for \( R = 0 \) and \( R \to \infty \) the insertion gain is infinite under asymptotic odd interface conditions. Practically we cannot realize asymptotic conditions, and that's the reason why the insertion gain is finite for \( R = 0 \) and \( R \to \infty \).

In the stopband we can see that insertion loss curves for R > 0 and especially for \( R > R_{\text{opt}} \), are close by the insertion loss curve for \( R \to \infty \) (filter without RC member). Therefore the equation (16) can be used for calculation the value of \( C_{\text{i}} \).

Measured insertion loss of the examined filter with \( R = R_{\text{opt}} \) at the frequency 100 kHz was 39.2 dB, Calculated insertion loss from the equation which can be easily evolved from (16) is 39.17 dB.

Experimental results show great congruity with calculated results.

Conclusions

Although we introduce RC members into RFI filters the insertion gain will exist, but it can be made as low as we want.

The elemental "L" section filter damped with an RC member can be designed analytically using equations from this paper.
Fig. 8: Measured insertion loss of the elemental "L" section filter damped with an RC mem-
ber (L = 2.2 mH, C, = 0.1 μF, C = 1 μF, R = variable)

Computer design is the most con-
venient method of designing more com-
plex, multiple "L" section filters.

From the practical example given in this paper great congruity between calculated and measured results can be seen.

References


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nents: State of the Art; New Deve-

