## PROGRESS IN FUZZY SETS AND SYSTEMS

edited by

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This volume contains selected contributions to the second Joint IFSA-EC EURO-WG Workshop on 'Progress in Fuzzy Sets in Europe' held in Vienna in 1988. The main emphasis in selecting the papers was on practical and theoretical relevance concerning the application and the development of this field of science. The reader, therefore will find both types of contributions in the book.

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# PROGRESS IN FUZZY SETS AND SYSTEMS

Edited by Wolfgang H. Janko, Marc Roubens and H.-J. Zimmermann



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## Introduction

This volume contains the proceedings of the Second Joint IFSA-EC and EURO-WGFS Workshop on *Progress in Fuzzy Sets in Europe* held on April 6 - 8, 1989 in Vienna, Austria.

The workshop was organized by Prof. Dr. Wolfgang H. Janko from the University of Economics in Vienna under the auspices of IFSA-EC, the European chapter of the International Fuzzy Systems Association, and EURO-WGFS, the working group on Fuzzy Sets of the Association of European Operational Research Societies. The workshop gathered more than 30 participants coming from Western European countries (Austria, Belgium, England, Germany, Finland, France, Hungary, Italy, Scotland and Spain) Eastern European countries (Bulgaria, the German Federal Republic, Hungary and Poland) and non-European countries such as China and Japan.

The 15 selected and refereed papers included in the volume are in principle the author's own versions, with limited editorial changes and small corrections. They are arranged in alphabetical order.

I wish to thank all the contributors for their valuable papers and an outstanding cooperation in the editorial project. I also would like to express my sincere thanks to Professor Dr. H. J. Zimmermann for the cooperation in the refereeing procedure.

Marc Roubens IFSA-EC and EURO-WGFS President

## FUZZY SET THEORY APPLICATION FOR FEEDFORWARD CONTROL OF ILL-DEFINED PROCESSES

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ABSTRACT. This article describes the principles of fuzzy feedforward control, suitable for feedforward control of ill-defined processes for which it is uneconomic or even impossible to develop a deterministic or stochastic process model and to apply the conventional feedforward control theory. Special attention is given to the experiments with fuzzy feedforward controllers and experimental results are presented.

#### 1. Introduction

One of the oldest principle in regulation is certainly that of invariant control, i.e. the achievement of total or partial independence of the controlled system under examination from disturbances acting upon it. Thousands of years ago the first devices for controlling windmills, constructed by the Arabs contained the elements of control for compensating the external load torque [1]. From that time till the modern age invariance control and especially feedforward control has been intensively studied and applied in numerous processes, but always on the basis of their deterministic or stochastic models.

For a certain number of processes, because of their complexity, limited knowledge or stochastic environment it is too expensive, or sometimes even impossible to develop a proper and valid deterministic or stochastic model and to apply the results of known, conventional feedforward control theory.

The fuzzy set theory has proved to be a suitable approach to modelling, simulation and control of these ill-defined systems. Many papers dealing with the application of the fuzzy set theory in feedback control of ill-defined systems have been published.

We have studied the disturbance decoupling and invariant control tasks, and used the methods of fuzzy set theory for feedforward control. With the proposed methods the area of application of the fuzzy control is widened to the invariant, feedforward control of illdefined processes and a new field of research is opened.

This paper review our research activities and presents new experimental results obtained with the fuzzy feedforward controller.

## Fuzzy models of ill-defined processes suitable for feedforward control

Several representations of fuzzy models of processes have been proposed as suitable for feedback control tasks. For the feedforward control another form of model has to be considered. Also in real situations process variables may have nominal operating values, or special control channels may be established to realize control. We have studied all these cases [2,3,4]. Only the case used in experiments (Ch,5) will be explained here.

Distinguishing the real space of manipulated input (U), disturbance input (D) and controlled output (Y) the fuzzy model of the first order process can be expressed in the form of the complex

$$M = \langle D, U, Y, S^* \rangle \tag{1}$$

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where S\* is a fuzzy relation defined by the fuzzy mapping of ordinary sets

$$S^*: D \times U \times Y \to Y \tag{2}$$

with membership function  $S^*(d,u,w,y)$ ,  $d\in D$ ,  $u\in U$ ,  $w,y\in Y$ . The fuzzy set  $y^*_{n+1}$  of the controlled output at a discrete time moment (n+1)T (T being the sampling interval) may be evaluated by the fuzzy relational equation

$$\mathbf{y}_{n+1}^* = \mathbf{S}^* \otimes \mathbf{u}_n^* \otimes \mathbf{d}_n^* \otimes \mathbf{y}_n^* \tag{3}$$

where  $u_n^*$ ,  $d_n^*$  and  $y_n^*$  are appropriate fuzzy sets of the manipulated input, disturbance input and controlled output in discrete time moment nT, and ® is a relational-relational composition operator.

If in the process description the principle of superposition can be applied then by considering process behavior near to the nominal operating values d<sub>N</sub>, u<sub>N</sub> and y<sub>N</sub>, the fuzzy model (1) can be changed into a form

$$M = \langle D, U, Y, P^*, R^* \rangle$$
 (4)

where  $P^*$  and  $R^*$  are fuzzy relations with membership functions  $P^*(\Delta d, \Delta y)$  and  $R^*(\Delta u, \Delta v)$  describing the relationship between the disturbance input increment  $\Delta d$  and the controlled output increment  $\Delta y$  and the relationship between the manipulated input increment  $\Delta u$  and the controlled output increment  $\Delta y$ .

The fuzzy set of the controlled output increment  $\Delta y^*_{n+1}$  due only to one of corresponding inputs may be evaluated by fuzzy relational equations

$$\Delta y_{n+1}^* = P^* \otimes \Delta d_n^*$$

$$\Delta y_{n+1}^* = R^* \otimes \Delta u_n^*$$
(6)

$$\Delta v_{n+1}^* = R^* \otimes \Delta u_n^* \tag{6}$$

 $\Delta d_n^*$  and  $\Delta u_n^*$  are appropriate fuzzy sets of disturbance and manipulated input increments. Fuzzy relations S\* or P\* and R\* can be obtained by one of the fuzzy identification procedures. In experiments we have used Pedrycz identification algorithm [5].

Equation (3) or equations (5) and (6) are essential parts of the fuzzy feedforward control algorithm.

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## 3. Fuzzy feedforward control algorithm

Two types of fuzzy feedforward control are distinguished. The first one is on-line fuzzy feedforward control. In each discrete time moment nT, the fuzzy set of control which will bring the process as close as possible to the total disturbance decoupling state, is calculated on-line using the fuzzy model of the process. In the second method the control procedure is based on the fuzzy feedforward controller relation calculated before (off-line).

Equation (3) or equations (5) and (6) are essential parts of on-line fuzzy feedforward control. Theoretically for the fuzzy system described with the equation (3) the total disturbance decoupling will be obtained if for each discrete time moment nT the fuzzy set of the controlled output  $y_n^*$  is equal to the fuzzy set of the controlled output in the previous time moment (n-1)T ,  $y_{n-1}^*$ , for all fuzzy sets of disturbance input. This means that in each time moment manipulated input ,  $u_n^*$ , must be calculated from the equation

$$y_n^* = S^* \otimes d_n^* \otimes y_n^* \otimes u_n^*$$

$$(7)$$

If in the description of the process behavior equations (5) and (6) can be used the starting equation is

$$P^* \otimes \Delta d_n^* = R^* \otimes \Delta u_n^* \tag{8}$$

because total disturbance decoupling will be obtained if the magnitude of the controlled output change  $\Delta y^*_{n+1}$  is the same for both changes of manipulated input  $\Delta u^*_n$  and disturbance input  $\Delta d^*_n$ .

The fuzzy feedforward control action is obtained by solving the equation (7) for  $u_n^*$  or solving the equation (8) for  $\Delta u_n^*$ . The problem is that such fuzzy relational equations have not a unique solution, or even worse there are a lot of cases when the exact solution doesn't exist. We have analyzed these cases and developed suitable algorithms for approximate, numerical solution of such equations [2,6,7].

## 4. Fuzzy feedforward controller

The fuzzy feedforward controller consists of three main parts: process interface, fuzzy interface and fuzzy feedforward control algorithm. Both process and fuzzy interface have input and output part as it is shown in Fig.1.

The process input interface consists of the measuring devices, but when the variables are scarcely measurable, the ability of the operator to observe and express linguistically the actual values of variables should be considered. At the fuzzy input interface these numeric or linguistic data are represented with fuzzy sets. The fuzzy output interface converts a fuzzy set of manipulated input or change of manipulated input into a single real value which represents the control action. Mean-value method or 'mean-of-maxima' can be used as an adequate technique in this interpretation procedure [2,6]. Through the output process interface this control action is applied to the process.

The principles of the fuzzy control algorithm are described in Ch.3. It is important to emphasis that in the case of the equation (8) the sign of the control action has to be determinate too. The control action will be minus, if the trends of the controlled output increment for the disturbance input only, and the trends of the controlled output increment for the manipulated input only are equal for the same trends of input variables, and vice

versa. If the trends of the controlled output increment are equal for the opposite trends of input variables, the control action will be plus.

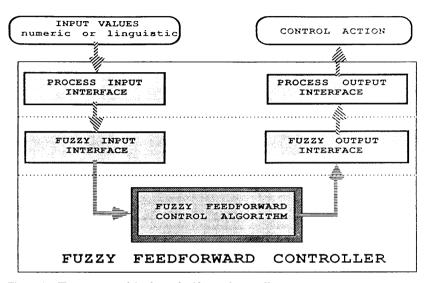


Figure 1. The structure of the fuzzy feedforward controller

After this short review of the fuzzy feedforward control principles, in the next section new experimental results obtained with fuzzy feedforward controller will be presented.

#### 5. Experiments with fuzzy feedforward controller

In the first part of our experimental research, which will be described here, the process was simulated on the analog computer and the fuzzy feedforward control algorithm was programmed on the 8 bit on-line control microcomputer system. The considered controlled process was the coupled tanks water level process structured according to Fig.2., which is often used as a "benchmark" problem. The input flow to the Tank 1 was chosen as a manipulated variable, the level of the Tank 2 as a disturbance variable and the level of the Tank 1 as a controlled output variable. The deterministic representation of the considered process is shown on Fig.2, also.

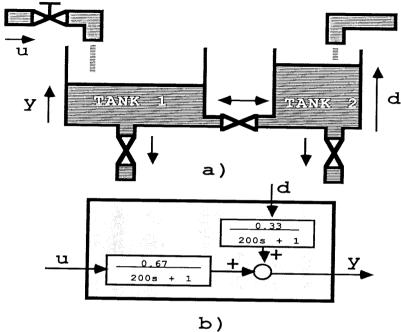


Figure 2. a) The coupled tanks water level process b) The deterministic representation of the considered process

Generally ,the first stage in the process control is determination of the objectives of control. In this case the task is to maintain the water in the Tank 1 at the constant level and make them independent of the level in the Tank 2.

The second stage is determination of the process model. Normal operating values of the process variables in these experiments were  $d_N = 5$ ,  $u_N = 5$  and  $y_N = 5$ . The fuzzy model of the process was established using 14 subjectively observed, non-accurate, cause-effect numeric data:

 $\{(\Delta d, \Delta y)\} = \{(-4.5, -0.18), (-2.1, -0.05), (-0.9, -0.015), (0.0), (1.3, 0.05), (3, 0.14), (5, 0.21)\}$  $\{(\Delta u, \Delta y)\}=\{(-4.5, -0.42), (-2, -0.22), (-1, -0.15), (0.0), (0.5, 0.1), (3, 0.28), (4.7, 0.43)\}$  The data were derived by human observation of variable values at the instruments where just zero, minimum and maximum were marked. Minimum (maximum) value of  $\Delta y$  was -0.5 (0.5) and of Δd and Δu -5 (5).

Each pair can be linguistically expressed with conditional statements, as for example the statement for pair  $(\Delta d, \Delta v) = (-0.45, -0.18)$  is

"If the water level in the Tank 2 decreases 4.5 units below nominal value,

then the water level in the Tank 1 will decrease 0.18 units 30 seconds later"

The fuzzy model of the process must be in the form of two fuzzy relations  $P^*(\Delta d, \Delta y)$ and  $R^*(\Delta u, \Delta y)$ . In order to unify the approach in such a way that it will be possible to use both numeric and linguistic data, a method of fuzzy discretisation of the real space is used [8]. It consists of cutting up the real space W into a series of non-disconnected fuzzy sets  $Z_1^*, Z_2^*, \dots, Z_1^*$ , such that the entire space is covered. Real value  $w_0 \in W$  now can be represented with the fuzzy vector

$$\underline{Z}_{w0}^* = [Z_1^*(w_0) \ Z_2^*(w_0) \ \cdot \cdot \cdot \ Z_I^*(w_0)]$$
 (9)

and every fuzzy set A\* which belongs to the space W and represents some linguistic data

can be represented with the fuzzy vector
$$\underline{Z}^*_{A^*} = [\sup(\min(Z^*_{I}(w), A^*(w))) \dots \sup(\min(Z^*_{I}(w), A^*(w)))] \quad (10)$$

Basic fuzzy sets  $Z_1^*$ , ...,  $Z_1^*$  of fuzzy discretisation used in these experiments are shown on Fig.3. Each pair  $(\Delta d, y)$  and  $(\Delta u, \Delta y)$  is now transformed into pairs of fuzzy vectors  $(\Delta d^*, \Delta y^*)$  and  $(\Delta u^*, \Delta y^*)$ . For example pair (-4.5, -0.18) is transformed into  $([0.7 \ 0.3 \ 0 \ 0 \ 0 \ 0]$ ,  $[0 \ 0.78 \ 0.92 \ 0 \ 0 \ 0]$ . The linguistic labels of basic fuzzy sets are given on Fig.3. also.

A Pedrycz algorithm [5] was used as an identification algorithm suitable for calculation of fuzzy relations  $P^*$  and  $R^*$ , which are now fuzzy matrices  $\underline{P}^* = [p_{\underline{lm}}]$  and  $\underline{R}^* = [r_{\underline{l}}]$ . The algorithm is based on the clustering technique and minimisation of the sum of distances between the output of the model derived calculated with equations (5) and (6) and the collected fuzzy output data  $\Delta y^*$ . The ISODATA algorithm [2] was applied as a clustering algorithm and the distance was specified as a Hamming distance. The minimal value of the performance index was obtained for seven clusters (each pair in its own cluster), and the final fuzzy matrices are

$$\mathbf{P}^* = [\mathbf{p}_{lm}] = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.08 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0.09 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.69 & 1 & 0.16 & 0.16 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$
(11)

$$\underline{R}^* = [r_{lj}] = \begin{bmatrix} 0.52 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.49 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.68 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.68 & 0.42 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.58 \end{bmatrix} \tag{12}$$

This fuzzy model can be easily converted into a linguistic model in which each rule has its degree of possibility. For example:

"Possibility that  $\Delta y$  is 'positive small' if  $\Delta d$  is 'positive big' is 0.8" "Possibility that  $\Delta y$  is 'negative big' if  $\Delta u$  is 'negative big' is 0.52"

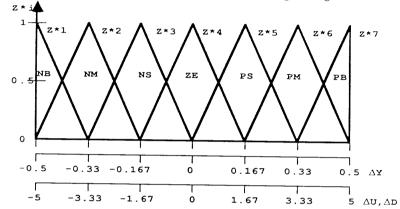


Figure 3. Basic fuzzy sets of fuzzy discretisation with their linguistic labels:

NB-negative big, NM-negative medium, NS-negative small, ZE-zero,
PS-positive small, PM-positive medium, PB-positive big

The 'max-min composition' was used in the fuzzy feedforward control algorithm. The fuzzy control  $\underline{\Delta u}^*$  was calculated either by  $\alpha$ -composition as the appropriate inverse composition of max-min composition, or by numerical methods, depending whether or not the equation (6) has the exact solution [2.6]. Let us illustrate the calculation procedure with  $\alpha$ -composition. For the fuzzy vector of disturbance input  $\underline{\Delta d}^* = [d_m]$  the element  $u_j$  of the fuzzy vector of control  $\underline{\Delta u}^* = [u_j]$  is obtained using the equation

$$u_{j} = \inf_{l} \{ \sup_{m} [\min_{l} (p_{lm}, d_{m})] \alpha r_{lj} \}$$
 (13)

where for  $a,b \in [0,1]$ 

$$\begin{array}{c} 1 \text{ , if } a \leq b \\ a \; \alpha \; b = \{ \\ b \text{ , if } a > b \end{array} \tag{14} \label{eq:14}$$

In the fuzzy output interface the 'mean-value method' was applied to represent the fuzzy vector of control with unique value from the interval  $\Delta U = [-5, 5]$ .

In order to compare the fuzzy approach with the conventional one, a linear regression model of the process is considered, too. For the same input-output data the regression model is

$$(\Delta y)_{d} = 0.0403 \cdot \Delta d \tag{16}$$

$$(\Delta y)_{ij} = 0.0953 \cdot \Delta u \tag{17}$$

Fig.4. shows the process response for the conventional feedforward control derived from this regression model and fuzzy feedforward control for pulse, cosine and saw-tooth disturbance. The ideal situation (absolute or total invariance) is a horizontal line. Advantages of the fuzzy approach are evident. The sampling interval was 19.2 seconds.

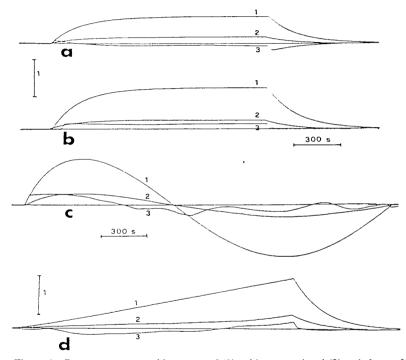


Figure 4. Process response without control (1), with conventional (2) and fuzzy (3) feedforward control for:

- (a) Pulse disturbance 1400 seconds long with 2.5 units amplitude,
- (b) Pulse disturbance 1400 seconds long with 4 units amplitude.
- (c) Cosine disturbance  $\Delta d = 5 \cdot \cos 0.002$ -t, and
- (d) Saw-tooth disturbance  $\Delta d = 0.0028 \cdot t$ . 1800 seconds long.

Fig.5. shows the process responses for the fuzzy feedforward control , saw-tooth disturbance and different sampling intervals. The control signal is shown, too. The change of the sampling interval to approximately 300 seconds has no great influence on the process response, but even for T=560 seconds the response is still satisfactory, although the magnitude of the control signal has been changed during the whole disturbance occurrence only four times .

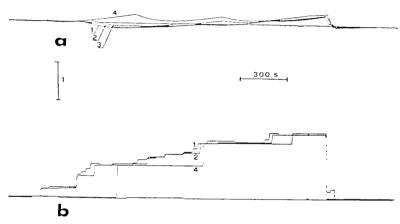


Figure 5. a) Process response for fuzzy feedforward control, saw-tooth disturbance and following sampling intervals: (1) T=19.2 s, (2) T=100 s, (3) T=300 s and (4) T=560 s (5) Control signals

The robustness of the fuzzy feedforward control to the variation of process fuzzy models parameters has been experimentally analyzed too. As an example Fig.6 shows process responses and a selection of corresponding control signals for the fuzzy feedforward control, saw tooth disturbance and various process fuzzy models. The values of the process fuzzy matrices  $\underline{p}^*$  and  $\underline{R}^*$  have been changed as follows:

- 1) Original form of  $\underline{P}^*$  and  $\underline{R}^*$  given with (11) and (12),
- 2) All values of  $\underline{P}^*$  increased 50 %,  $\underline{R}^*$  unchanged.
- 3) All values of  $\underline{R}^*$  increased 50 %,  $\underline{P}^*$  unchanged,
- 4) All values of  $\underline{P}^*$  and  $\underline{R}^*$  increased 50 %.

5) All nonzero values of  $\underline{P}^*$  and  $\underline{R}^*$  changed to one. This means replacement of fuzzy matrices with Boolean matrices



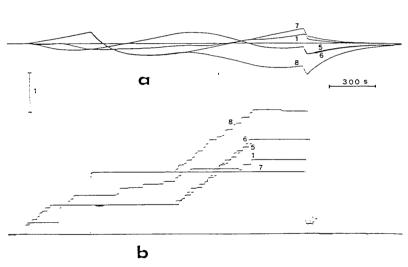


Figure 6. a) Process response for fuzzy feedforward control, saw-tooth disturbance and: (1) original fuzzy marrices  $\underline{P}^*$  and  $\underline{R}^*$ , (2)  $\underline{P}^*$  increased 50%,  $\underline{R}^*$  unchanged, (3)  $\underline{P}^*$  unchanged,  $\underline{R}^*$  increased 50%, (4)  $\underline{P}^*$  and  $\underline{R}^*$  increased 50%, (5),(6) (7) and (8)  $\underline{P}^*$  and  $\underline{R}^*$  Boolean matrices (see text)

b) Selected control signals

- 6) All values of  $\underline{P}^*$  and  $\underline{R}^*$  bigger or equal to 0.1 replaced with 1 and values less then 0.1 replaced with 0,
- 7) All values of  $\underline{P}^*$  and  $\underline{R}^*$  bigger or equal to 0.4 replaced with 1 and all values less then 0.4 replaced with 0,and
- 8) In each row of  $\underline{P}^*$  and  $\underline{R}^*$  only the biggest value is replaced with 1 and all other values are set to 0. This means that in each row there is just one non zero element. Exception is the third row of matrix  $\underline{P}^*$ , because in its original form there are two values equal to 1. The matrix  $\underline{R}^*$  is now the unity matrix.

The process responses show that the fuzzy feedforward control algorithm is quite flexible as regards changes to the process fuzzy model. For the cases 1), 2) and 3) the response is almost the same as in original case. When the fuzzy model is replaced with the Boolean model the process response is not so good as before, but it is still satisfactory, especially for cases 5) and 6). The Boolean model only describes the process behavior roughly. In identifying such a model it is sufficient to evaluate and linguistically express the possible effects of the disturbance variable changes to the process output variables and of the manipulate variable changes to the process output variable, without giving their degrees of possibility. For example some of the conditional statements for the Boolean model are:

"If the increment of the disturbance input variable (Δd) is 'positive big', then the output increment (Δy) will be somewhere between 'positive small" and 'positive medium' "

or

"If the increment of the manipulated input variable ( $\Delta u$ ) is 'negative small' then the output increment ( $\Delta y$ ) will be also 'negative small'

As a result of further process model simplification (case 7. and particularly case 8.) the worse process response is obtained according to the process response obtained for basic Boolean model (case 5.), especially for higher values of the disturbance signal.

Influence of various inverse relational-relational composition operators used in the algorithm for solving fuzzy relational equations has been also analysed. Process responses shown in Fig.7. correspond to the saw tooth disturbances, original algorithm where  $\alpha$ -composition is used as an inverse composition (eq.13) and modified algorithm where  $\alpha$ -composition is replaced with max-min composition. For this second case the element  $u_j$  of the control vector  $\Delta u^*$  was calculated using the equation

$$u_{j} = \sup_{l} \{ \min[\sup_{m} (\min(p_{lm}, \Delta d_{m})), r_{lj}] \}$$
 (19)

When it was not possible to calculate u<sub>j</sub> from the equation (13) or (19) the algorithm for approximate, numerical solution of fuzzy relational equations [6] was used.



Figure 7. Process response for saw tooth disturbance and fuzzy feedforward control with (1) α-composition and (2) max-min composition used for solving fuzzy relational equations

In both cases process responses were quite satisfactory. Max-min composition gave even better results for higher values of disturbance signals, although according to the fuzzy mathematics  $\alpha$ -composition must be used as an inverse composition for solving fuzzy relational equations with max-min composition [9].

Analyzing the experiment results we can conclude that in the same conditions, which means the same vague, non-accurate input-output data used in process model identification, the fuzzy feedforward approach gives better results then the conventional feedforward approach. Another advantage of the fuzzy feedforward control is controller robustness to the changing of the sampling interval, to the changing the process fuzzy model and to the choice of the inverse relational-relational composition operator used in algorithm for solving fuzzy relational equations.

#### 6. Conclusion

The proposed and develop methods of fuzzy feedforward control are rather simple and effective, useful and applicable in all situations when only subjectively interpreted, inaccurate and linguistically expressed information about process behavior or disturbance variables values are known.

Experiments with a fuzzy feedforward controller shows that it gives better results then the conventional feedforward controller derived from the same data about the process behavior. Also the fuzzy feedforward controller is fairly robust to the changing of the sampling interval and the process fuzzy model parameter variations. Also the choice of the inverse relational-relational composition operator used in the algorithm for solving fuzzy relational equations is not so critical.

At the end it should be emphasis that the fuzzy feedforward control is not a substitute for the conventional feedforward control. It is only an alternative approach for invariant , disturbance decoupling control of ill-defined processes where conventional methods don't give the desired results.

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