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Fuzzy feedforward and composite control

by D. Stipaničev*, MSc, Dipl Ing, and Prof Dr J. Božičević†, Dipl Ing

The concept of fuzzy feedforward and composite control of complex, ill-defined, non-deterministic processes is offered. Fuzzy relational model of process suitable for feed-forward control is developed and, using this model, the algorithms of the fuzzy feedforward and composite controls are derived. The advantages of proposed control procedures are demonstrated by example of the invariant fuzzy control of an irrigation process.

Nomenclature

e	error of controlled variable
$E(\cdot)$	mean value
I, J, K, L	sets of indices
M	fuzzy model of process
Q	distance between fuzzy sets
R	relation between controlled output and appropriate input
T	sampling interval
u	manipulated variable
y	controlled variable
Δ	increment
μ	membership grade
ρ	distance
ω	disturbance variable
χ, ν, ζ	fuzzy sets for the partition of real spaces
E, U, Y, Ω	real spaces

Subscripts

c	number of clusters
C	composite control
d	delay
E	Euclidean
FD	feedback control
FF	feedforward control
H	Hamming
N	nominal operating value
k, n	instant of time

notation of vectors and matrices in bold face

Superscripts

$*$	fuzzy quantity
$\sim$	approximate quantity
$\hat{}$	upper band
L	linguistic quantity

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1 Introduction

In process control, the achievement of the total or partial independence of the controlled system under examination with respect to the disturbances acting upon it is one of the most important control goals (Lyben, 1973).

In the case of a so-called 'invariant' control, the feed-forward control and composite (feedforward-feedback) control have been approved as a better solution than the feedback alone. Until now they have been studied and applied in numerous processes on the basis of deterministic models (Lyben, 1973; Mandić and Božičević, 1985). For a great number of industrial processes, their complexity, limited knowledge or stochastic environment prevent us from giving a precise, unique and reliable description of input-output relationship and developing a proper deterministic model.

In recent years, the use of the fuzzy set theory has proved a favourable approach to modelling and control of such ill-defined, non-deterministic processes. Many papers dealing with the construction, modification and analysis of the fuzzy controllers and their application to the feedback control have been published (for example, Mamdani, 1977; King and Mamdani, 1977; Braae and Rutherford 1979b; Czogala and Pedrycz, 1982). We have studied the feed-forward tasks and used the methods of linguistic modelling and fuzzy control as a new basis of feedforward and composite control (Božičević, 1983; Stipaničev and Božičević, 1985).

This paper presents a new approach to fuzzy feed-forward and fuzzy composite control, the control procedures being developed by means of the fuzzy relational models of processes. We would like to stress that we do not suggest the substitution for conventional feedforward and composite control where it can be applied, but an alternative approach for those situations where conventional methods fail.

2 Fuzzy models of processes for feedforward control

2.1 Background

A brief survey of basic knowledge necessary to describe fuzzy models of processes is given as an introduction. More details can be found in literature (for example, Dubois and Prade, 1980).

Let W be an ordinary set, and μ_{A^*} a function which assigns to each element w in W a number $\mu_{A^*}(w)$ in the interval [0; 1]. Then the function μ_{A^*} specifies a *fuzzy set* A^* of the set W. The function μ_{A^*} is called the membership function of the fuzzy set A^*, and its value $\mu_{A^*}(w)$ describes the grade of membership of the element w in A^*. The set W is often called the *universe of discourse*.

Let $W_1, W_2, \ldots W_k$ be ordinary sets. The function μ_{R^*}, which assigns to each element $(w_1, w_2, \ldots, w_k)$ in the

Cartesian product $W_1 \times W_2 \times \ldots \times W_k$ a number $\mu_{R^*}(w_1, w_2, \ldots, w_k)$ in the interval [0; 1] specifies a *fuzzy relation* R^* of the set $W_1 \times W_2 \times \ldots \times W_k$.

Let A^* and B^* be fuzzy sets of a set W. The *union* ($\cup$) and *intersection* ($\cap$) of fuzzy sets A^* and B^* are defined as

$$C^* = A \cup B \Leftrightarrow \forall w \in W: \mu_{C^*}(w) = \max [\mu_{A^*}(w), \mu_{B^*}(w)] \qquad \ldots(1)$$

$$D^* = A \cap B \Leftrightarrow \forall w \in W: \mu_{D^*}(w) = \min [\mu_{A^*}(w), \mu_{B^*}(w)] \qquad \ldots(2)$$

Let A^* be the fuzzy set of the set W_1, B^* and C^* the fuzzy sets of the set W_2 and R^* the fuzzy relation of the set $W_1 \times W_2$. *Max-min* ($\bigcirc$) and *α-composition* (@) of the set A^* and fuzzy relation R^* are defined as

$$B^* = R^* \bigcirc A^* \Leftrightarrow \forall w_2 \in W_2: \mu_{B^*}(w_2) = \sup_{w_1 \in W_1} [\min(\mu_{R^*}(w_1, w_2), \mu_{A^*}(w_1))] \qquad \ldots(3)$$

$$C^* = R^* \,@\, A^* \Leftrightarrow \forall w_2 \in W_2: \mu_{C^*}(w_2) = \inf_{w_1 \in W_1} [\mu_{R^*}(w_1, w_2) \alpha \mu_{A^*}(w_1)] \qquad \ldots(4)$$

where

$$\mu_{R^*}(w_1, w_2) \alpha \mu_{A^*}(w_1) = \begin{cases} 1, \textit{iff}\ \mu_{R^*}(w_1, w_2) \leq \mu_{A^*}(w_1) \\ \mu_{A^*}(w_1), \textit{iff}\ \mu_{R^*}(w_1, w_2) > \mu_{A^*}(w_1) \end{cases} \qquad \ldots(5)$$

The *fuzzy discretisation* (Malvache and Willaeys, 1981) is a method which makes possible the use of fuzzy and non-fuzzy forms of information. It consists of cutting up a real space W (the universe of discourse) into a series of non-disconnected fuzzy sets $Z_1^*, Z_2^*, \ldots, Z_I^*$ such that the entire space W is covered:

$$\forall_{w \in W} \ \exists_{1 \leq i \leq I} : \mu_{Z_i^*}(w) > 0 \qquad \ldots(6)$$

Thus every fuzzy and non-fuzzy set which belongs to space W can be represented by fuzzy vectors in terms of Z_i^*. For example, the real value $w_0 \in W$ can be represented by

$$\mathbf{Z}^* = [\mu_{Z_1^*}(w_0) \mu_{Z_2^*}(w_0) \ldots \mu_{Z_I^*}(w_0)] \qquad \ldots(7)$$

and the fuzzy set $A^* \subset W$ by

$$\mathbf{Z}^* = \{\sup_{w \in W} [\![\min [\mu_{Z_1^*}(w), \mu_{A^*}(w)]]\!] \ldots \sup_{w \in W} \times [\![\min [\mu_{Z_I^*}(w), \mu_{A^*}(w)]]\!]\} \qquad \ldots(8)$$

2.2 Fuzzy models

Several representations of fuzzy models of processes have been proposed (Willaeys and Malvache, 1978 and 1982), being suitable for feedback control only. For the feedforward control tasks another form of model has to be considered.

Distinguishing the real space of manipulated input (U), disturbance input (Ω) and controlled output (Y), the fuzzy model of the k-th order process can be expressed in the form of the complex

$$M = \{Y, U, \Omega, R^*\} \qquad \ldots(9)$$

where R^* is a fuzzy relation describing the relationship between a sequence of the controlled output in discrete time moments $nT, (n+1)T, \ldots, (n+k-1)T$, manipulated input applied in the moment $[(n+k)T - t_{du}]$, disturbance input applied in the moment $[(n+k)T - t_{d\omega})$ and controlled output in discrete time moment $(n+k)T$. t_{du} and $t_{d\omega}$ are delays of the manipulated input-controlled output and disturbance input-controlled output paths of the process. Relation R^* is of the $k+2$ order and is defined on the Cartesian product $U \times \Omega \times \underbrace{Y \times \ldots \times Y}_{k \text{ times}}$.

If in the process description the principle of the superposition can be applied, the fuzzy model (9) can be changed to a form

$$M = \{Y, U, \Omega, R_U^*, R_\Omega^*\} \qquad \ldots(10)$$

where R_U^* and R_Ω^* are fuzzy relations describing the relationship between the sequence of the controlled output and manipulated input and the relationship between the sequence of the controlled output and disturbance input.

We shall continue to deal with such processes; only, for the sake of simplicity, we shall concentrate on the first-order processes, using the controlled output increment as a process output variable instead of the controlled output only. The results can be easily extended to the processes of higher order.

In this case the fuzzy relations are $R_U^*(u, y, \Delta y)$ and $R^*(w, y, \Delta y)$. The fuzzy set Δy^* of the controlled output increment, due only to one of the corresponding inputs, may be evaluated by the fuzzy relational equations:

$$\Delta y^* = R_U^* \bigcirc y^* \bigcirc u^* \qquad \ldots(11)$$

$$\Delta y^* = R_\Omega^* \bigcirc y^* \bigcirc \omega^* \qquad \ldots(12)$$

$u^* \subset U$, $\omega^* \subset \Omega$ and $y^* \subset Y$ are appropriate fuzzy sets of manipulated input, disturbance input and controlled output. Max-min composition is denoted by $\bigcirc$.

Process variables usually have nominal operating values y_N, ω_N and u_N. Considering process behaviour in the vicinity of these values, fuzzy relations can be specified and defined as $R_U^*(\Delta u, \Delta y)$ and $R_\Omega^*(\Delta\omega, \Delta y)$, where Δu and $\Delta\omega$ are changes of the manipulated input and disturbance input from their nominal operating values. The fuzzy relations (11) and (12) are in this case

$$\Delta y^* = R_U^* \bigcirc \Delta u^* \qquad \ldots(13)$$

$$\Delta y^* = R_\Omega^* \bigcirc \Delta\omega^* \qquad \ldots(14)$$

Fuzzy relations R_U^* and R_Ω^* can be obtained by any of the suitable fuzzy identification procedures (Pedrycz, 1982; Pedrycz, 1984; Higashi and Klir, 1984). A procedure considered in this paper is adapted for the cases in which the process variables have nominal operating values. It will be presented in a few words.

In order to unify the approach and make use of the fuzzy and non-fuzzy forms of information available, a method of fuzzy discretisation is applied. The fuzzy partition of real spaces ΔY, ΔU and $\Delta\Omega$ into a series of fuzzy sets $\{\chi_l^*\}$, $\{\nu_j^*\}$ and $\{\zeta_m^*\}$ is used, such that every point of ΔY, ΔU and $\Delta\Omega$ 'belongs' to at least one of the χ_l^*, ν_j^* and ζ_m^*. An example of fuzzy sets χ_l^*, ν_j^* and ζ_m^* of input and output space is shown in Fig 1, together with their linguistic labels. Now the fuzzy relations R_U^* and R_Ω^* have matrix representation $\mathbf{R}_U^* = [r_{lj}]$, $\mathbf{R}_\Omega^* = [p_{lm}]$ where r_{lj} gives the possibility measure of χ_l^* with respect to ν_j^* and p_{lm} gives the possibility measure of χ_l^* with respect to ζ_m^*. For example, using the linguistic labels of the sets χ_l^*, ν_j^* and ζ_m^* from Fig 1, $r_{32} = 0.93$ means: "If the increment of the manipulated input is negative medium, the possibility of the increment of the controlled output being negative small is 0.93".

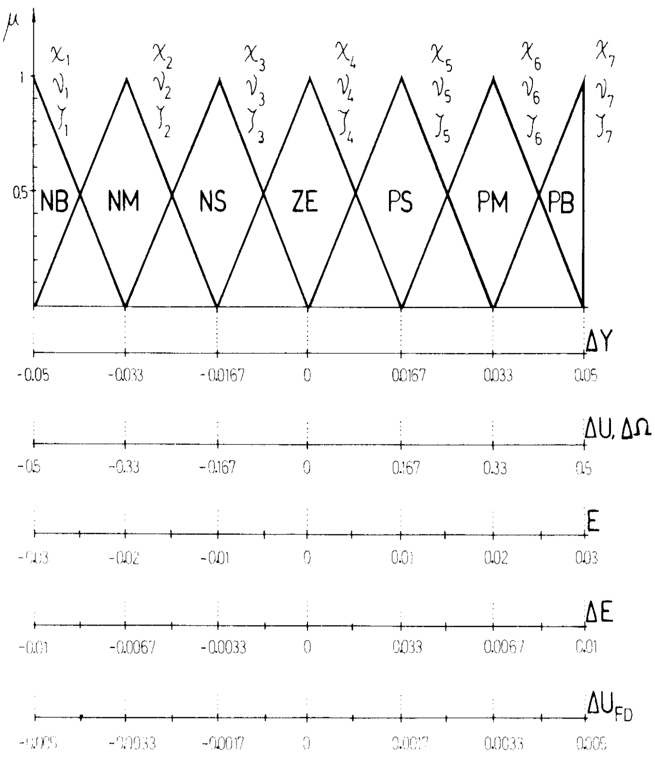

Fig 1 Input-output data, basic fuzzy sets of fuzzy discretisation with its linguistic labels, and definition of fuzzy sets for variables e, Δe and Δu_{FD} of the fuzzy feedback controller (NB – negative big, NM – negative medium, NS – negative small, ZE – zero, PS – positive small, PM – positive medium, PB – positive big)

Two procedures for the collection of input–output data can be distinguished. In the first, the experience and knowledge of operators and/or experts are used. Such a description of the process behaviour is usually qualitative linguistic description in the form of the cause-effect conditional statements. For example: "If the disturbance input is slightly below nominal operating value, then the controlled output increment, after the process time delay, will be moderate to fairly negative".

Defining the fuzzy language (Zadeh, 1971), to each linguistic value of process variables adequate fuzzy set is assigned. Using Eqn (8), the collections of fuzzy vectors in terms of series of fuzzy sets $\{\chi_i^*\}$, $\{\nu_j^*\}$ and $\{\zeta_m^*\}$ can be obtained. They are input information for fuzzy identification.

In the second procedure, the real cause-effect values $(\Delta u, \Delta y)$ and $(\Delta\omega, \Delta y)$ are collected and transformed with Eqn (7) to appropriate fuzzy vectors. As in the first approach, these fuzzy vectors are input information for fuzzy identification. The combination of both approaches may be used.

The final result is two sets of input–output fuzzy vectors $(\Delta\mathbf{u}_{k1}^*, \Delta\mathbf{y}_{k1}^*)$, $k1 = 1, \ldots, K1$ and $(\Delta\omega_{k2}, \Delta\mathbf{y}_{k2})$, $k2 = 1, \ldots, K2$. The task of the fuzzy identification can be stated as follows:

Define the matrices $\mathbf{R}_U^*$ and $\mathbf{R}_\Omega^*$ that satisfy the set of the fuzzy relational equations

$$\Delta\mathbf{y}_{k1}^* = \mathbf{R}_U^* \bigcirc \Delta\mathbf{u}_{k1}^*, \quad k1 = 1, \ldots, \quad K1 \qquad \ldots(15)$$

$$\Delta\mathbf{y}_{k2}^* = \mathbf{R}_\Omega^* \bigcirc \Delta\omega_{k2}^*, \quad k2 = 1, \ldots, \quad K2 \qquad \ldots(16)$$

where for each $k1$ and $k2$ $(\Delta\mathbf{u}_{k1}^*, \Delta\mathbf{y}_{k1}^*)$ and $(\Delta\omega_{k2}^*, \Delta\mathbf{y}_{k2}^*)$ are collected input–output pairs.

The simplest and ideal situation is when each of the pairs $(\Delta\mathbf{u}_{k1}^*, \Delta\mathbf{y}_{k1}^*)$ and $(\Delta\omega_{k2}^*, \Delta\mathbf{y}_{k2}^*)$ satisfies the corresponding fuzzy relational Eqn (15) or (16). According to Sanchez (1976) the greatest fuzzy relations $\hat{\mathbf{R}}_U^*$ and $\hat{\mathbf{R}}_\Omega^*$, such that fuzzy equations are valid, can be calculated by the formulas:

$$\hat{\mathbf{R}}_U^* = \bigcap_{k1=1}^{K1} (\Delta\mathbf{u}_{k1}^* @ \Delta\mathbf{y}_{k1}^*) \qquad \ldots(17)$$

$$\hat{\mathbf{R}}_\Omega^* = \bigcap_{k2=1}^{K2} (\Delta\omega_{k2}^* @ \Delta\mathbf{y}_{k2}^*) \qquad \ldots(18)$$

In practical applications, the assumption that each of the pairs perfectly fits the fuzzy relational equations will be not valid. For those situations the Pedrycz identification algorithm is suitable (Pedrycz, 1984), being based on the clustering technique and minimisation of the sum of distances between the output of the model described by means of the fuzzy relational equations and collected fuzzy output data. This identification procedure for fuzzy relation R_U^* is:

(a) Division of a set of pairs $(\Delta\mathbf{u}_{k1}^*, \Delta\mathbf{y}_{k1}^*)$ into c clusters $(1 \leqslant c \leqslant K1)$.

(b) Characterisation of the elements belonging to the same n-th cluster C_n by mean value vectors

$$\mathbf{E}(\Delta\mathbf{u}_n^*) = [E(\Delta u_j)_n] \quad \text{and} \quad \mathbf{E}(\Delta\mathbf{y}_n^*) = [E(\Delta y_l)_n]$$

$$E(\Delta u_j)_n = \sum_{C_n} (\Delta u_j)_{k1} / N_n \qquad \ldots(19)$$

$$E(\Delta y_l)_n = \sum_{C_n} (\Delta y_l)_{k1} / N_n \qquad \ldots(20)$$

where the sum is taken over all fuzzy sets belonging to the n-th cluster, while N_n is equal to the number of elements contained in the n-th cluster.

(c) Calculation of performance index for the various numbers of clusters $c\ (1 \leqslant c \leqslant K1)$:

$$Q_c = \sum_{k1=1}^{K1} \rho\ [(\widetilde{\mathbf{R}}_U^*)_c \circ \Delta \mathbf{u}_{k1}^* ; \Delta \mathbf{y}_{k1}^*] \qquad \ldots(21)$$

where

$$(\widetilde{\mathbf{R}}_U^*)_c = \bigcap_{n=1}^{c} [\mathrm{E}(\Delta \mathbf{u}_n^*) @ \mathrm{E}(\Delta \mathbf{y}_n^*)] \qquad \ldots(22)$$

and ρ is a Minkowski distance (for example, Hamming distance)

(d) Search for a number of clusters $c_{\min}$ which minimise the performance index Q_c

$$c_{\min}:\ Q_{c_{\min}} = \min_{1 \leqslant c \leqslant K^1} Q_c \qquad \ldots(23)$$

This procedure gives the approximation of the fuzzy relation $\widetilde{\mathbf{R}}_U^* = (\widetilde{\mathbf{R}}_U^*)_{c_{\min}}$ which best satisfies the fuzzy relational Eqn (15).

The same procedure is used for a fuzzy relation $\widetilde{\mathbf{R}}_\Omega^*$. Fuzzy relations so obtained are essential parts of the fuzzy feedforward control algorithm.

3 Fuzzy feedforward control

The fuzzy feedforward control loop is designed as shown in Fig 2. The controller 'observes' the disturbances and the output of the process. In spite of its composite structure, the control unit is the feedforward controller, because the feedback data are only the information on the dynamic properties of the process.

Figure 3 shows the structure of the fuzzy feedforward controller. It is conceived as an assembly of units which perform the interface functions and produce the fuzzy feedforward control actions.

The units for input and output interface permit the flow of information between the process and the computer which controls the process according to the fuzzy control algorithm. Each interface unit consists of two parts:

(a) process input and output interface, and
(b) fuzzy input and output interface.

The process input interface consists of measuring devices; but, when the variables are scarcely measurable (as, for example, colour), the ability of an operator to observe and express linguistically the actual values of variables may be considered. In the latter case fuzzy sets of input variables are defined by means of fuzzy language (Zadeh, 1971).

In the fuzzy input interface these input data (in a form of real numbers or fuzzy sets) are represented according to fuzzy discretisation method by the fuzzy vectors. For

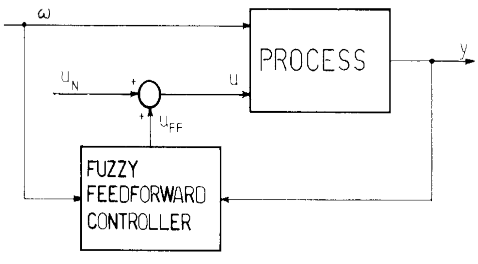

Fig 2 The fuzzy feedforward control loop

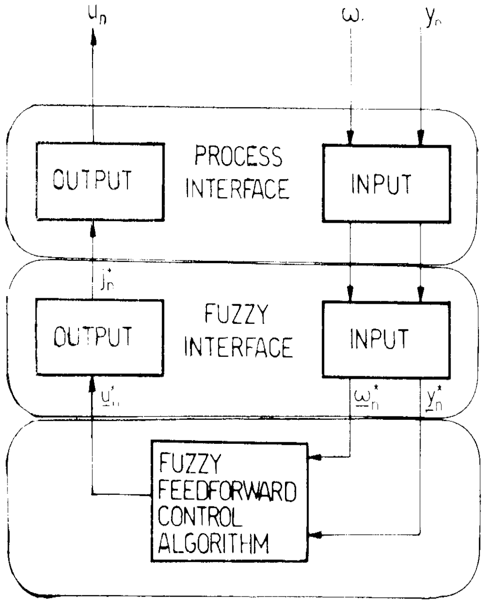

Fig 3 Structure of the fuzzy feedforward controller

example, if the change of disturbance input is $\Delta\omega = 0.2$ and the basic fuzzy sets of fuzzy discretisation are given in Fig. 1, the fuzzy vector representing this real value is $\Delta\omega^* = [0\ 0\ 0\ 0\ \ 0.8\ \ 0.2\ \ 0]$. It is important to observe that both the fuzzy sets defined to represent linguistic values of variables observed by the operator and the basic fuzzy sets of fuzzy discretisation are not generally the same.

The fuzzy output interface converts an output fuzzy vector into a single value which presents the control action. We use for this conversion the 'mean value method' by means of which the output is calculated as a mean value of the elements in the output fuzzy set. For example, if the output fuzzy set is $\mathbf{u}^* = [u_1 \ldots u_j \ldots u_J]$, its single value representation will be

$$j^+ = \frac{\sum_{j=1}^{J} j \cdot u_j}{\sum_{j=1}^{J} u_j} \qquad \ldots(24)$$

J is the number of basic fuzzy sets of output fuzzy discretisation.

The process output interface associates this control action with a real value, which is the adequate control command to the process. The association is performed according to

$$u = (j^+ - 1)\frac{u_{\max} - u_{\min}}{J - 1} + u_{\min} \qquad \ldots(25)$$

where $u_{\max}$ and $u_{\min}$ are the minimal and maximal values of manipulated input used in fuzzy discretisation.

The fuzzy feedforward control action is formulated by means of one on-line and two off-line procedures. The control action trend and the control action delay are determined by the off-line procedures, while the control action magnitude is evaluated by the on-line procedure.

The control action trend will be minus if the trends of the controlled output increment for the disturbance input only and of the controlled output increment for the manipulated input only are equal for the same trends of input variables, and vice versa. If the trends of the controlled output increments are equal for the opposite trends of the input variables, the control action trend will be plus.

The values of delays in the controlled output response for the manipulated input (t_{du}) and in the controlled output response for the disturbance input ($t_{d\omega}$) must be estimated, at least roughly, as the basis for the determination of the control action delay. The controller acts in the instant of time $t = t_i + \tau$, where t_i is the instant of the occurrence of disturbances and τ is the difference of delay times, $\tau = t_{d\omega} - t_{du}$. The fuzzy feedforward controller will act successfully, as all other feedforward controllers do, only if process response to the disturbance is equal or slower than that to the manipulated action.

The procedure for the evaluation of the control action magnitude in the instant of time nT has two steps:

(1) Calculate the fuzzy vector Δy_n^* of the increment of the controlled output, due to the actual disturbance input and controlled output in the instant of time nT, using the Eqn (12). The values of the actual disturbance input controlled output are expressed by fuzzy vectors ω_n^* and $\mathbf{y}_n^*$

(2) Considering the evaluated fuzzy vector $\Delta \mathbf{y}_n^*$ and observed $\mathbf{y}_n^*$, solve fuzzy Eqn (11) for $\mathbf{u}_n^*$. This fuzzy vector $\mathbf{u}_n^*$ defines the control action magnitude and it is the input information of the fuzzy output interface.

The solution of fuzzy Eqn (11) is the most troublesome part of this procedure. If the solution exists, the least upper bound $\hat{\mathbf{u}}_n^*$ of all solutions can be calculated by the equation (Sanchez, 1976):

$$\hat{\mathbf{u}}_n^* = (\mathbf{R}_U^* \bigcirc \mathbf{y}_n^*) @ \Delta \mathbf{y}_n^* \qquad \ldots(26)$$

In applications there are cases when $\mathbf{R}_U^*$, $\mathbf{y}_n^*$ and $\Delta \mathbf{y}_n^*$ are such that the exact solution does not exist. Then the numerical approach is more convenient. Pedrycz has developed a numerical method for solving fuzzy equations (Pedrycz, 1982) by searching the fuzzy set $\tilde{\mathbf{u}}_n^*$ which minimises the Euclidean distance between the fuzzy sets $\Delta \mathbf{y}_n^*$ and $\Delta \tilde{\mathbf{y}}_n^* = \mathbf{R}_U^* \bigcirc \mathbf{y}_n^* \bigcirc \tilde{\mathbf{u}}_n^*$.

In our approach we have modified this procedure and have obtained the following advantages:

(i) short computation
(ii) easier interpretation of the resulting fuzzy set of action, $\mathbf{u}_n^*$, and
(iii) possibility of small control microcomputer application.

The basis of the procedure proposed in this paper is the use as a resulting fuzzy set of action either the fuzzy set $\hat{\mathbf{u}}_n^*$ obtained with Eqn (26) or one of J degenerated fuzzy sets:

$$(\bar{\mathbf{u}}_n^*)_j = [\bar{u}_i]; \quad \bar{u}_i = \begin{cases} 0, & \text{iff} \quad i \neq j \\ 1, & \text{iff} \quad i = j \end{cases} \qquad \ldots(27)$$

The criterion of selection is the minimisation of the modified Hamming distance between $\Delta \mathbf{y}_n^*$ and $\Delta \hat{\mathbf{y}}_n^* = \mathbf{R}_U^* \bigcirc \mathbf{y}_n^* \bigcirc \hat{\mathbf{u}}_n^*$ or Δy_n^* and $\Delta \bar{\mathbf{y}}_n^* = \mathbf{R}_U^* \bigcirc \mathbf{y}_n^* \bigcirc (\bar{\mathbf{u}}_n^*)_j$, $j = 1, \ldots, J$. The reason for the modification of Hamming distance is the fact that, in the process control, the absolute difference between approximate fuzzy sets $\Delta \hat{\mathbf{y}}_n^*$ or $\Delta \bar{\mathbf{y}}_n^*$ and the exact fuzzy set $\Delta \mathbf{y}_n^*$ have to be equally considered as the difference between their interpreted values.

The first step in the procedure is the estimation of the element j^+ of the support set of the fuzzy set $\mathbf{u}_n^*$ which minimises the modified Hamming distance. Let

$$\Delta \mathbf{y}_n^* = [\Delta y_l], \quad l \in \mathbf{L}, \quad \mathbf{L} = \{1, \ldots, L\}$$

$$\mathbf{y}_n^* = [y_i], \quad i \in \mathbf{I}, \quad \mathbf{I} = \{1, \ldots, I\}$$

$$\mathbf{u}_n^* = [u_j], \quad j \in \mathbf{J}, \quad \mathbf{J} = \{1, \ldots, J\}$$

$$\mathbf{R}_U^* = [r_{ilj}]$$

where L, I and J are the numbers of basic fuzzy sets of fuzzy discretisation, and

$$\mathbf{R}_{An}^* = [a_{lj}] = \mathbf{R}_U^* \bigcirc \mathbf{y}_n^*.$$

If the set

$$\mathbf{K} = \{k' \mid Q(k') = \inf_k Q(k)\} \qquad \ldots(28)$$

where for $k \in \mathbf{J}$

$$Q(k) = \begin{cases} \sum\limits_{l \in \mathbf{L}} |l - l_{\text{int}}| \cdot |a_{lk} - \Delta y_l|, & \text{iff } l_{\text{int}} \notin \mathbf{L} \\ \sum\limits_{\substack{l \in \mathbf{L} \\ l \neq l_{\text{int}}}} |l - l_{\text{int}}| \cdot |a_{lk} - \Delta y_l| + |a_{l_{\text{int}} k} - \Delta y_{l_{\text{int}}}|, & \text{iff } l_{\text{int}} \in \mathbf{L} \end{cases} \qquad \ldots(29)$$

and

$$l_{\text{int}} = \sum_{l \in \mathbf{L}} l \cdot \Delta y_l / \sum_{l \in \mathbf{L}} \Delta y_l \qquad \ldots(30)$$

has only one element $k' = k^+$, then $j^+ = k^+$, else j^+ may be any one element k' of the set $\mathbf{K}$. In some cases a reasonable criterion exists which makes possible the evaluation of a unique solution. For example, if the fuzzy set $\mathbf{u}_n^*$ refers to the fuel consumption $\mathbf{K} = \{6, 7\}$, and the linguistic labels of 6th and 7th basic fuzzy set of output fuzzy discretisation are 'large' and 'very large', then $j^+ = 6$ (linguistic label large) could be chosen to save the fuel.

The second step in the procedure is the calculation of the distance

$$Q(J+1) = \begin{cases} \sum\limits_{l \in \mathbf{L}} |l - l_{\text{int}}| \cdot |\Delta \hat{y}_l - \Delta y_l|, & \text{iff } l_{\text{int}} \notin \mathbf{L} \\ \sum\limits_{\substack{l \in \mathbf{L} \\ l \neq l_{\text{int}}}} |l - l_{\text{int}}| \cdot |\Delta \hat{y}_l - \Delta y_l| + |\Delta \hat{y}_{l_{\text{int}}} - \Delta y_{l_{\text{int}}}|, & \text{iff } l_{\text{int}} \in \mathbf{L} \end{cases} \qquad \ldots(31)$$

where

$$\Delta \hat{y}_l = \sup_{j \in \mathbf{J}} [\min(\hat{u}_j, a_{lj})], \qquad \ldots(32)$$

$\hat{\mathbf{u}}_n^* = [\hat{u}_j]$ is obtained with Eqn (26) and l_{int} with Eqn (30).

Now, if $Q(j^+)$ is less than $Q(J+1)$, the resulting fuzzy set $\mathbf{u}_n^*$ is $(\bar{\mathbf{u}}_n)_{j^+}$, otherwise $\mathbf{u}_n^* = \hat{\mathbf{u}}_n^*$.

Figure 4 shows a flow chart of the fuzzy feedforward control algorithm.

4 Fuzzy composite control

The feedforward control based on the fuzzy model cannot ensure the highest possible degree of the process

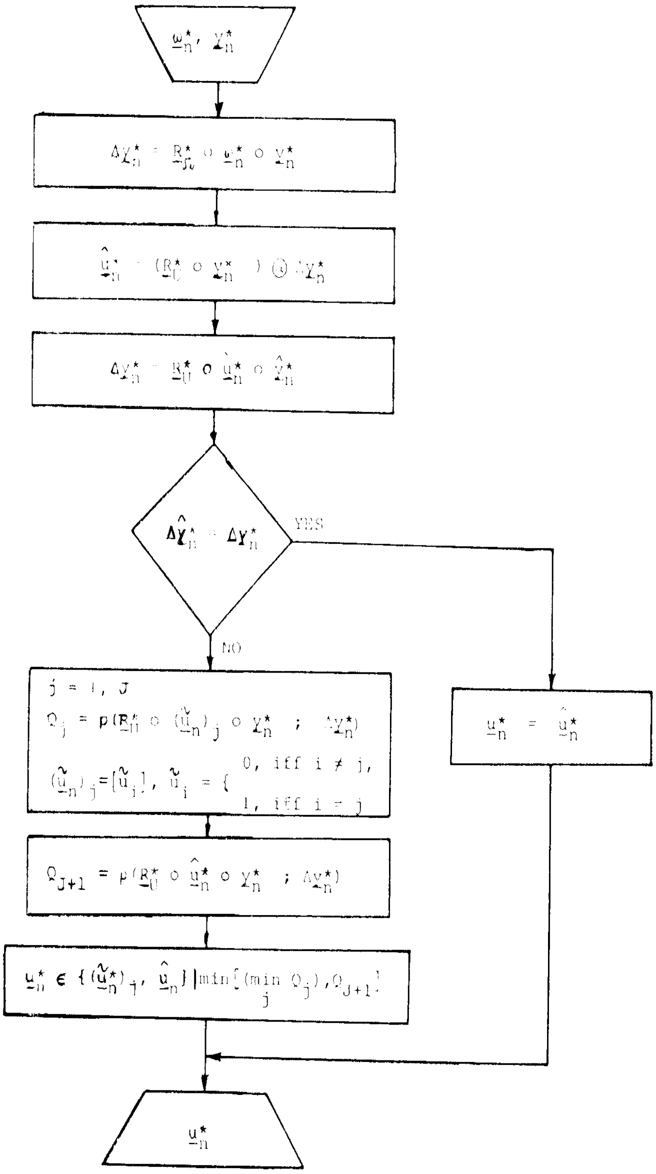

Fig 4 The fuzzy feedforward controller algorithm

invariance to disturbances, not only because of the fuzzy model imperfection but also because of the influence of the unobservable disturbances. To overcome this problem and to improve the degree of the process invariance, the feedback controller is added (Luyben, 1973), and composite (feedforward–feedback) control is obtained.

The fuzzy composite control loop is shown in Fig 5. The controller 'observes' simultaneously the disturbances and the process output. Now the process output data are information for the feedback action. The feedforward and feedback part of the fuzzy composite controller act independently and each has its own input and output interface.

The proposed procedure is used for the fuzzy feedforward control signal, u^*_{FFn}. The fuzzy feedback control signal u^*_{FDn} is calculated by the fuzzy equation

$$u^*_{FDn} = F^*_{FD} \bigcirc e^*_n \bigcirc \Delta e^*_n \qquad \ldots(33)$$

where F^*_{FD} is the fuzzy relation $F^*_{FD}(e_n, \Delta e_n, u_{FDn})$ on $E \times \Delta E \times U$ of the fuzzy feedback controller, and E and ΔE are real spaces of the output error and its increment. The fuzzy set of the controlled output error (e^*_n) and its increment (Δe^*_n) may be obtained either by the operator's observation or by calculation and fuzzification from the actual values of the controlled output, its previous and desidered values. Fuzzy relation F^*_{FD} may be obtained by any of the existing synthesis methods for the fuzzy feedback controllers (for example, Mamdani, 1977; Braae and Rutherford, 1979a; Czogala and Pedrycz, 1982).

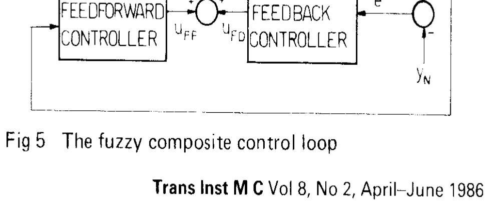

Fig 5 The fuzzy composite control loop

The real values of the feedforward and feedback signals, u_{FFn} and u_{FDn} are evaluated in appropriate fuzzy and process output interfaces. The resulting composite control action is

$$u_{Cn} = u_{FFn} + u_{FDn} \qquad \ldots(34)$$

5 An illustrative example

The application of the proposed fuzzy feedforward and composite control procedures is illustrated by a simple numerical example.

It is necessary to maintain water in certain paddy fields at a constant level to ensure maximum yield from the rice crop. Water is supplied to the paddy fields from a reservoir high in the neighbouring mountains and its flow is controlled by a floodgate. The flow from the lake in the vicinity is the disturbance over which we have no control, but we can measure the level of the lake which directly depends on this disturbance flow.

Figure 6 illustrates the situation schematically. The deterministic representation of the considered process is also shown because the given example is performed by computer simulation. In reality, instead of this deterministic model, the set of cause-effect numeric and/or linguistic data is only known.

For convenience, let us suppose that nominal operating values of the process variables are $\omega_N = 0.5$, $u_N = 0.5$ and $y_N = 0.5$ and that collected data set of the input-output pairs are those displayed in Table 1 (sampling interval 5 min). Some of these data can be linguistically interpreted with the following statements:

(a) "If the level of the neighbouring lake increases 0.3 units, the level of the paddy field will start to rise after approximately 25 min, having increased 0.014 units 5 min later".

(b) 'If the floodgate is closed so that the flow decreases 0.2 units, the level of the paddy field will start to fall after approximately 20 min, having decreased 0.022 units 5 min later".

TABLE 1: Pairs of input-output data

$\Delta\omega$	Δy	Δu	Δy
−0.45	−0.018	−0.45	−0.042
−0.21	−0.0051	−0.20	−0.022
−0.09	−0.0015	−0.10	−0.015
0	0	0	0
0.13	0.005	0.05	0.010
0.30	0.014	0.30	0.028
0.50	0.021	0.47	0.043

The delays in the manipulated input-controlled output response t_{du} and in the disturbance input-controlled output response $t_{d\omega}$ are troublesome from the control point of view. Basic fuzzy sets, $\chi_l^*, \nu_j^*, \zeta_m^*$, of input $(\Delta U, \Delta\Omega)$ and output (ΔY) spaces are chosen arbitrarily and are presented in Fig. 1, together with their linguistic labels. Transforming $(\Delta\omega, \Delta y)$ and $(\Delta u, \Delta y)$ on fuzzy data, using Eqn (7), we get pairs of fuzzy vectors $(\Delta\boldsymbol{\omega}^*, \Delta\mathbf{y}^*)$ and $(\Delta\mathbf{u}^*, \Delta\mathbf{y}^*)$ collected in Table 2.

The ISODATA algorithm (Dubois and Prade, 1980) was applied, as a clustering algorithm, the distance ρ being specified as Hamming distance. The minimal value of the performance index Q [Eqn (21)] is for $c = 7$ (each pair in its own cluster). Fuzzy matrices $\mathbf{R}_\Omega^*$ and $\mathbf{R}_U^*$ are obtained according to Eqn (22) and are as follows:

$$\mathbf{R}_\Omega^* = [p_{lm}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.078 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0.09 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.695 & 1 & 0.162 & 0.162 & 0 \\ 0 & 0 & 0 & 0 & 0.3 & 1 & 0.796 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.204 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{R}_U^* = [r_{lj}] = \begin{bmatrix} 0.52 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.49 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.68 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.68 & 0.42 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.58 \end{bmatrix}$$

This fuzzy model can be easily converted into a linguistic model, in which each rule has its degree of possibility.

TABLE 2: Pairs of input-output fuzzy sets

$\Delta\omega^*$							Δy^*						
0.7	0.3	0	0	0	0	0	0	0.078	0.922	0	0	0	0
0	0.253	0.737	0	0	0	0	0	0	0.305	0.695	0	0	0
0	0	0.54	0.46	0	0	0	0	0	0.09	0.91	0	0	0
0	0	0	1	0	0	0	0	0	0	1	0	0	0
0	0	0	0.222	0.778	0	0	0	0	0	0.7	0.3	0	0
0	0	0	0	0.198	0.802	0	0	0	0	0.162	0.838	0	0
0	0	0	0	0	0	1	0	0	0	0	0.796	0.204	0
Δu^*							Δy^*						
0.7	0.3	0	0	0	0	0	0.52	0.48	0	0	0	0	0
0	0.198	0.802	0	0	0	0	0	0.32	0.68	0	0	0	0
0	0	0.6	0.4	0	0	0	0	0	0.9	0.1	0	0	0
0	0	0	1	0	0	0	0	0	0	1	0	0	0
0	0	0	0.7	0.3	0	0	0	0	0	0.4	0.6	0	0
0	0	0	0	0.198	0.802	0	0	0	0	0	0.32	0.68	0
0	0	0	0	0	0.18	0.82	0	0	0	0	0.42	0.48	0

TABLE 3: Set of linguistic rules of fuzzy feedback controller

Δe^L \ e^L	PB	PM	PS	ZE	NS	NM	NB
PB	NB	NB	NB	NB	NM	NS	ZE
PM	NB	NB	NB	NM	NM	NS	ZE
PS	NB	NB	NM	NM	NS	ZE	PM
ZE	NB	NM	NS	ZE	PS	PM	PB
NS	NM	ZE	PS	PM	PM	PB	PB
NM	ZE	PS	PM	PM	PB	PB	PB
NB	ZE	PS	PM	PB	PB	PB	PB
	Δu^L_{FD}						

Some of the rules of the linguistic model are as follows:

(a) "Possibility that the output is *PS* if the disturbance input is *PB* is 0.796."
(b) "Possibility that the output is *NB* if the manipulated input is *NB* is 0.52."

For fuzzy feedback control, the Andersen–Nielsen method (Andersen and Nielsen, 1985) in non-adaptive form is used. Table 3 gives the set of linguistic rules of the applied fuzzy feedback controller. The real spaces E, ΔE and ΔU_{FD} are represented by 13 discrete points and the appropriate fuzzy sets are given in Fig 1.

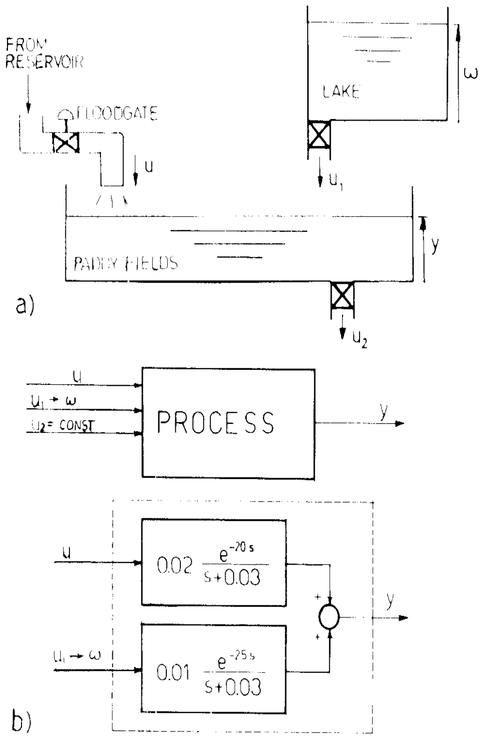

Fig 6 (a) The water level process. (b) The deterministic representation of the considered process (time delays and time constants in minutes)

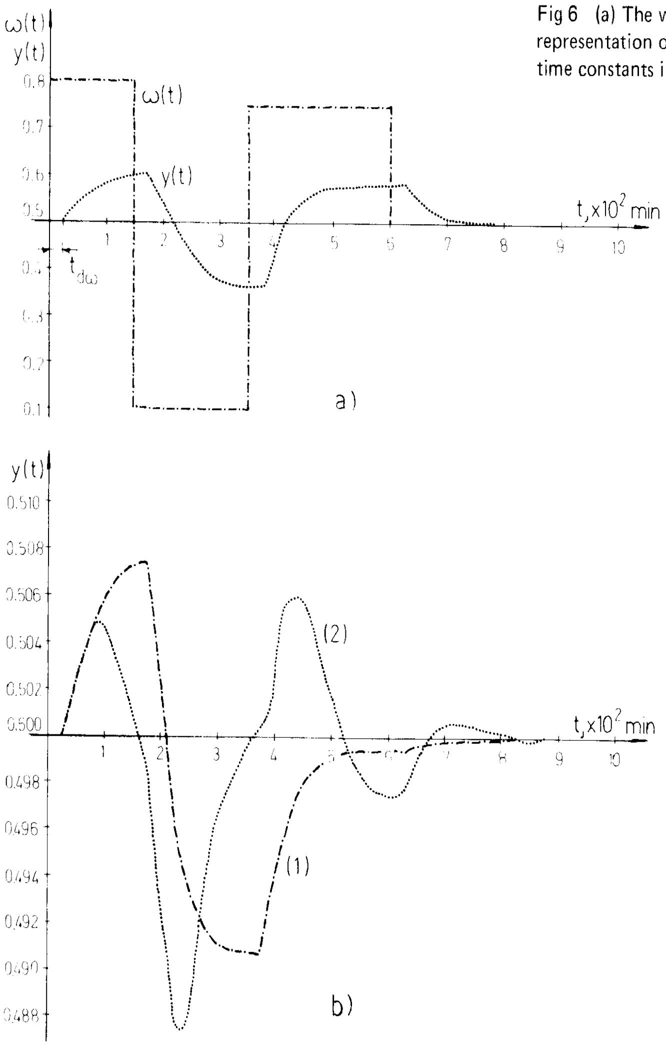

Fig 7 (a) Disturbance input $\omega(t)$ and process response $y(t)$. (b) Process response on the same disturbance $\omega(t)$ when fuzzy feedforward (1) or fuzzy composite (2) control is applied

The fuzzy feedforward and fuzzy composite control actions are compared in Fig 7, using the step input. The sum of absolute errors

$$\text{SAE} = \sum_{n=1}^{200} (y_n - y_N)$$

are $\text{SAE}_{FF} = 54.65$ and $\text{SAE}_C = 43.89$.

In order to compare the fuzzy approach with the conventional approach, a linear regression model is considered. For input-output data from Table 1, the regression model presents $\Delta y = 0.0403\,\Delta\omega$ and $\Delta y = 0.0953\,\Delta u$. The conventional feedforward controller derived from this model gives $\text{SAE}_{FF} = 158.08$, while the composite controller with PID unit in the feedback part gives $\text{SAE}_C = 93.85$. The PID feedback unit of the conventional composite controller was tuned for the same decay ratio which was obtained with the fuzzy feedback controller.

6 Conclusion

There are many real situations in industry where fuzzy control is effectively applied. Since the first work in this field was published in 1974, there are many theoretical and experimental studies on fuzzy control, but they have been primarily concerned with various feedback control tasks.

This paper presents a method of fuzzy reasoning for both feedforward and composite (feedforward-feedback) controls and shows how to derive fuzzy control actions when, as a control criterion, the degree of the process invariance to disturbances is considered.

A new method of the fuzzy feedforward control has been developed and presented, being based both on the fuzzy relational model of the process and the theory of fuzzy relational equations. Using the proposed method, adequate control results can be obtained even when only a rough and subjectively interpreted information on the process behaviour is attainable.

A numerical example is performed to illustrate the described methods. The process fuzzy relational model is developed with a few input-output data, and with seven basic fuzzy sets used in the fuzzy discretisation. The algorithm improves the invariance of the considered process in relation to the conventional feedforward and composite control (with PID controller in the feedback part), in spite of the use of the simple model.

At the end, it is important to emphasise that this fuzzy approach to feedforward and composite control is in no way in contradiction with the conventional, deterministic or stochastic one. It is only a complementary approach which enables application of the principles of invariant automatic control to the situations where conventional methods do not give desired results.

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