FUZZY-LINGUISTIC MODELS FOR FEEDFORWARD CONTROL

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ABSTRACT

The idea of fuzzy feedforward control is presented as a new approach to the control of complex processes, particularly suitable when a high degree of invariance has to be achived.
The fuzzy-linguistic models of the process has been proved as a favorable base for feedforward control. The linguistic model of the process is developed and a fuzzy feedforward controller is designed. The quality of the fuzzy feedforward controller is evaluated through the quantitative and qualitative measure.
Selected nonlinear process illustrates the proposed idea.

KEYWORDS: feedforward control, fuzzy sets, linguistic modelling

1. INTRODUCTION

The conventional control of numerous industrial processes has not given desired quality, either because of their complexity an of limited knowledge to develop proper deterministic model or because of the stohastic environment. The manual control since these days has been only possible solution. Recentrly, the use of linguistic models of such processes have been proven as faworable approach to the solution of automatic control problem |1|.

Feedforward control is much more efficient then the feedback control when a high degree of invariance to external disturbances has to be achieved. External disturbances are undesirable changes of process input variables. The feedforward control make possible effective reduction of the upset caused by disturbances, even in the cases when the approximative model of the process is known |2|. During last two decades it has been studied more intensively and applied on numerous processes |2|. The deterministic models has been used as the basis of control. Then the method of linguistic modelling and fuzzy control are developed as the new basis of feedforward control |3|.

Two approaches to the application of linguistic modelling in control are in use. In the first approach the ability and experience of the human operator in controlling the process is studied and the linguistic model which describes linguisticallyhow the operator controls the process is developed |1|. Such a control algorithm is the copy of the human-operator control behaviour, and it can be imroved only by trial-and-error method. In the second approach the knowledge of the operator or an experimentator about the controlled process is used |4,5,6|. The control algorithm can be obtained explicitly from the linguistic model by the metnod of linguistic synthesis |4|, from the fuzzy model via satisfaction function of criteria |5|, or via a performance index |6|. Now the trial-and-error method is transformed to a sort of the identification procedure requied to produce adequate linguistic model of the process. The quality of model depends on the tiral-and-error method, but if the fuzzy identification |7| is applied this dependance sholud be considerably overcome.

The new approach to feedforward control presented in this paper is based on the application of the linguistic model of the process. Particular attention is given to the structure of such a model and to the generation of linguistic rules for feedforward controller.

2. THE NEW APPROACH TO THE FEEDFORWARD CONTROL

The fuzzy feedforward control loop is designed according to Fig. 1.

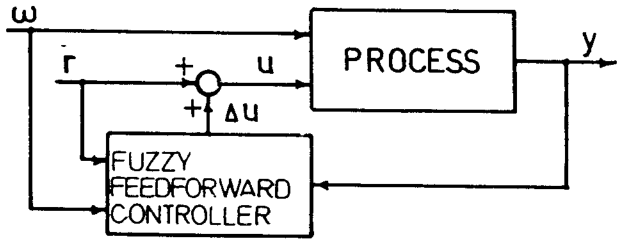


Fig. 1. The fuzzy feedforward control loop

The controller observes the disturbances and the output of the process. In spite of such composite structure it is really feedforward controller. The output data are information on the nonlinear properties of the process only.

The structure of the fuzzy feedforward controller is given in Fig. 2.

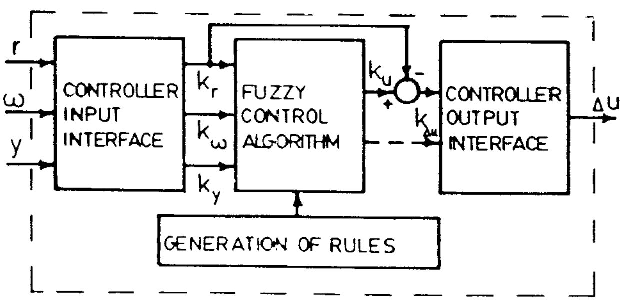


Fig. 2. The structure of the fuzzy feedforward controller



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The fuzzy feedforward controller is conceived as an assembly of units which performs the interface functions, generation of the linguistic rules and fuzzy control. It is necessary to distinguish real (e.g. $\omega$, $u_o$, 5), linguistic (e.g. $u_L$, $\omega_{LR}$, "NEGATIV BIG") and fuzzy (e.g. $\Omega^*$, $u_U(\bar{u})$) variables and values.

The units for input and output interface permit flow of information beetwen the process and computer which control the process according the fuzzy control algorithm. The representation of real physical variables must take the values on the finite sets of integers, $K_v = \{1,\ldots,K_v\}$, (v can be any real variable e.g. $\omega$, y, u) and therefore the discrete intervals method is used . The fuzzy control algorithm gives a non-fuzzy value of the manipulated input variable, $k_u$, or its increment, $k_{\Delta u}$.

The first step to the synthesis of the proposed controller is the generation of the linguistic rules. It is based on the linguistic model of the controlled process.

## 3. LINGUISTIC MODEL FOR FEEDFORWARD CONTROL

Static or dynamic linguistic model may be used.

In static linguistic model the relations between variables are given in the form of the conditional statements. For example:

"If the controlled variable is positive big, manipulated variable is zero and disturbance is negative small, the next steady-state value of the controlled variable will be positive medium".

The conditional statement represents one linguistic rule of the linguistic model. The complete model consists of the set of such rules $R_L=\{r_{L,i} \mid i=1,\ldots,N\}$. Each rule connects linguistic values of the input and output linguistic variable. Linguistic variables usually take values on the finite sets of terms $T_v$, where v can be any one linguistic variable.

We have supposed the representation of the static linguistic model by means of linguistic equation. It is linguistically described static characteristic of the process.

$$\underline{y}_{LN} = F(\underline{y}_L, \underline{u}_L, \underline{\omega}_L) \qquad (1)$$

where $\underline{y}_{LN}$, $\underline{y}_L$, $\underline{u}_L$ and $\underline{\omega}_L$ are linguistic vectors of the controlled variables, manipulated variables and disturbance variables respectively, and $\underline{F}$ is linguistic matrix function usually expressed in tabular form.

The linguistic model (1) is the model of the multi input-multi output system (MIMO), but it may be decupled into M models of multi input-single output systems (MISO), where M is the order of the linguistic vector $\underline{y}_{LN}$ and $\underline{v}_L$. We shall deal only with MISO models with one manipulated and one disturbance input, although the results can be easily applied to the general case. Static linguistic model may be also presented as

$$\Delta y_{LN} = \underline{F}(y_L, u_L, \omega_L) \qquad (2)$$

where $\Delta y_{LN}$ is incremental form of controlled variable. When the process is linear, equation (2) can be reduced to

$$\Delta y_{LN} = \underline{F}(u_L, \omega_L) \qquad (3)$$

because $\Delta y_{LN}$ does not depend about the output $y_L$.

In dynamic linguistic models the rule describes the dynamic behaviour of the process in the continous or discrete form with the conditional statement. For example:

"If the controlled variable is high, manipulated variable small and disturbance medium, then the controlled variable will decrease slowly"

or

"If at the instant, nT, the controlled variable is very big, the manipulated variable is small and the disturbance medium, then at the instant, (n+1)T, the controlled variable will be medium to big".

The complet set of rules, $R_L$, gives the linguistic description of the low of evolution of the process output variable.

The discrete dynamic linguistic model of the process is generally represent by linguistic equations

$$\underline{x}_{L_{n+1}} = \underline{F}(\underline{x}_{L_n}, \underline{u}_{L_n}, \underline{\omega}_{L_n}) \qquad (4.a)$$

$$\underline{y}_{L_n} = \underline{G}(\underline{x}_{L_n}, \underline{u}_{L_n}, \underline{\omega}_{L_n}) \qquad (4.b)$$

$\underline{x}_L$ is the linguistic vector of the state-space variables.

Simple class of model

$$\underline{x}_{L_{n+1}} = \underline{F}(\underline{x}_{L_n}, u_{L_n}, \omega_{L_n}) \qquad (5.a)$$

$$y_{L_n} = \underline{G}(\underline{x}_{L_n}) \qquad (5.b)$$

is considered in this paper. This is the state space dynamic linguistic model of the process. It is difficult to obtaine the linguistic model in the form (5) if the process is of the higher order. The linguistic identification procedure becomes too complicated, and therefore the design of feedforward controller by means of transfer dynamic linguistic model of the process is more suitable. It has form

$$y_{L_{n+1}} = \underline{F}(y_{L_n}, u_{L_n}, \omega_{L_n}) \qquad (6)$$

Usually process variable have normal operating values $Y_{LR}$, $U_{LR}$ and $\Omega_{LR}$ and then more convinient form of the model is

$$y_{L_{n+1}} = \underline{F}(y_{L_n}, \Delta u_{L_n}, \Delta\omega_{L_n}) \qquad (7)$$

This equation gives the dependance of the controlled variable and the manipulated and disturbance incremental variables in the vicinity of the normal operating values.

The described models are the basis for the generation of the linguistic rules for the fuzzy feedforward controller.

## 4. GENERATION OF LINGUISTIC RULES FOR THE FUZZY FEEDFORWARD CONTROLLER

The rules have to contain the instructions how to carry on the control procedure. If the linguistic reference value of the disturbance input, $\omega_L$, is $\Omega_{LR}$, the desired static linguistic model of the process is

$$\Delta y_{LN} = \underline{F}(y_L, r_L, \omega_L = \Omega_{LR}) \qquad (8)$$

and desired transfer dynamic linguistic model in discrete form is

$$y_{L_{n+1}} = \underline{F}(y_{L_n}, r_{L_n}, \omega_{L_n} = \Omega_{LR}) \qquad (9)$$

or

$$y_{L_{n+1}} = \underline{F}(y_{L_n}, \Delta r_{L_n}, \Delta\omega_{L_n} = \text{ZE}) \qquad (10)$$

where ZE means "zero". Equations (8), (9) and (10) are reference linguistic models of the undisturbed processed and therefore the variables, $u_L$ and $\Delta u_L$, were substituted with reference input manipulated variables, $r_L$ and $\Delta r_L$.

Although the problem of invariance is most important in the feedforward control, the purposed structure makes possible other control goals. For example the desired process response. It can be specified with linguistic equations of the same form as (8), (9) and (10) with substituing $\underline{F}_D$ instead $\underline{F}\cdot\underline{F}_D$ is the linguistic function which describes new, desired characteristics of the process.

Let us start with the generation of linguistic rules for the static fuzzy feedforward controller. The linguistic equation of the control is

$$u_L = \underline{F}_C(y_L,\ r_L,\ \omega_L) \qquad (11)$$

The procedure of linguistic synthesis consists of

(i) the solution of the equation (8) and

(ii) the linguistic inversion of the equation (2).

For specific values of $y_L$ and $r_L$ the equation (8) is solved and corresponding value of $\Delta y_L$ is found. Then for that value of $\Delta y_L$, and the specific value of $\omega_L$, the search of $\underline{F}(y_L, u_L, \omega_L)$ gives the value $u_L$ that satisfy the equation $\Delta y_L = \underline{F}(y_L, u_L, \omega_L)$, so the value of $\underline{F}_C(y_L, r_L, \omega_L)$ is defined. Then the procedure has to be rebeted for all values of $y_L$, $r_L$ and $\omega_L$.

The function $\underline{F}_C$, obtained by described procedure, is usually incomplete and multy-valued and must be completed with the adequat criteria for approximation (see section 5). In the case of the single-valued form of $\underline{F}_C$ the specific criteria should be defined, too.

If the process is linear, then the linguistic equation of the control is

$$u_L = \underline{F}_C(r_L,\ \omega_L) \qquad (12)$$

and it can be obtained more easily.

Linguistic equation of the dynamic fuzzy feedforward control is

$$u_{L_{n+1}} = \underline{F}_C(y_{L_n},\ r_{L_n},\ \omega_{L_n}) \qquad (13)$$

or

$$u_{L_{n+1}} = \underline{F}_C(y_{L_n},\ \Delta r_{L_n},\ \Delta\omega_{L_n}) \qquad (14)$$

The procedure of the linguistic synthesis of the set of the rules is the same as for the static controller.

The linguistic rules of the controller ($R_L$ is the set of these rules) can not be directly applied in control procedures. They have be transformed into the fuzzy form and after that fuzzy feedforward control algorithm can be defined.

The transformation of the linguistic rules into the fuzzy form requires definition of fuzzy language that assigns meaning to the elements of the set of terms |8|. Our stand-point is that for practical purposes the most convinient method for definition of the fuzzy language is the standard form of fuzzy sets $\mu_\delta(i)$ |9|. Then each rule can be represented with four positive numbers. Such a method permits easy modification of the rules and implementation of the control algorithm in any computer system, even on small micro-processor system.

Disturbances, reference input and the controlled output are usually nonfuzzy. In such a case the membership function of the fuzzy set $U^*$ or $\Delta U^*$ of the manipulated variable can be easilly determinated by the Andersen-Nielsen method |9|. The nonfuzzy value of the manipulated variable $k_U$ or $k_{\Delta U}$ is obtained by one of the interpretation methods (mean of maxima, modified mean of maxima etc.).

The proposed fuzzy feedforward controller can act sucesfully only if the response of the process on the disturbance is equal or slower then the response of the process on the manipulated action. The values of delay have to be estimated, at least roughly, in advance, because the controller must act in the moment $t=t_i+\tau$, where $t_i$ is the moment of the occurance of the disturbance and $\tau$ is the difference of dealy times, $\tau=t_{d\omega}-t_{du}$.

## 5. PERFORMANCE INDICES FOR THE FUZZY FEEDFORWARD CONTROLLER

Various performance indices could be set as a measure of behaviour of a fuzzy feedforward controller. We have propose the quantitative and the qualitative index of performance.

The first index is the measure of the controller incompletness. Let $R_L^+$ is the set of the controller linguistic rules, $r_L^+$, which can not be found by searchnig procedure ($R_L^+ \subseteq R_L$). The rules are approximated. The measure of controller incompletness, $M_I$, is then defined as a ratio of the cardinality of the set, $R_L^+$, and the cardinality of the set, $R_L$.

$$M_I = \frac{\text{card } R_L^+}{\text{card } R_L} \qquad (15)$$

If controller is ideally designed $M_I=0$ ($R_L^+=\emptyset$), while if the design is completelly deficient $M_I=1$ ($R_L^+=R_L$).

Another performance index is the linguistic measure of controller behaviour. Let us suppose that the equation (8) describes the desired model. The linguistic substitution of the equation (11) into the equation (2) gives

$$y_L = \underline{F}(y_L, \underline{F}(y_L, r_L, \omega_L), \omega_L) = \underline{F}_O(y_L, r_L, \omega_L) \qquad (16)$$

where $\underline{F}_O$ is the obtained linguistic function. Let the linguistic measure of controller behaviour be applied as the performance index and defined with the equations

$$\bar{Q} = \max_{L\ y_L, r_L, \omega_L} Q(y_L, r_L, \omega_L) \qquad (17)$$

$$Q(y_L, r_L, \omega_L) = d_L(\underline{F}(y_L, r_L, \omega_L=\Omega_{LR}), \underline{F}_O(y_L, r_L, \omega_L)) \qquad (18)$$

where $\max_L$ is the linguistic operation of maximum, and $d_L$ is the linguistic conformity function. $d_L$ takes values on the finite set of terms $T_{dL}$. The set $T_{dL}$ has to be strictly partialy ordered in the linguistic sense, because the linguistic operation of maximum has to be performed on its elements.

If the linguistic maximum of $d_L(\underline{F}, \underline{F}_O)$ exists, it will be easy to prove that it is evaluated for the values of $y_L$, $r_L$ and $\omega_L$, which on the another side define incomplet rules, $r_L^+$, of the set $R_L^+$. Therefore the linguistic minimization of $d_L(\underline{F}, \underline{F}_O)$ may be completed when the approximative value of $u_L$ is reached. For the ideal case ($M_I=0$) the set $r_L^+$ is empty and the maximal $\bar{Q}$ will not exist. All $Q(y_L, r_L, \omega_L)$ are equal to the element of $T_{dL}$ which expresses the best behaviour of the controller.

Few factors has influence to the quality of the fuzzy feedforward control action

(i) the behaviour of the fuzzy feedforward controller,

(ii) the adequacy of the model of the process, and,

(iii) the measurability of the disturbances.

The control action of the fuzzy feedforward controller may be judged by means of the conventional indices (e.g. error, integral square error etc.).

## 6. THE EXAMPLE

The presented fuzzy feedforward control is illustrated with the simple example.

Nonlinear first order process is supposed and structured according Fig. 3.

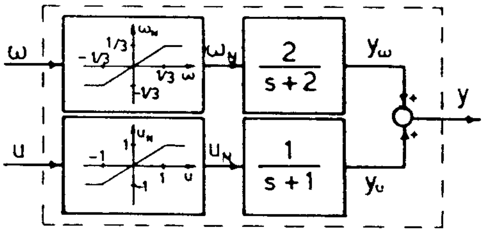


Fig. 3. Block diagram of the nonlinear first order process

The elements of the sets of terms $T_{uL}$, $T_{yL}$ and $T_{\Delta yL}$ are: negative big (NB), negative medium (MN), negative small (NS), zero (ZE), positive small (PS), positive medium (PM) and positive big (PB), and the elements of the set of terms $T_{\omega L}$ are: negative (N), zero (ZE) and positive (P).

Static linguistic model of this process represents equation (2) nad is expressed by Tables 1., 2. and 3.

Let Table 2. express the desire linguistic model of the process, and that $u_L$ is replaced with $r_L$. The set of rules of the static fuzzy feedforward controller is then obtained using the linguistic synthesis method and the linguistic equation of the controlle is expressed by Table 4., 5. and 6.

For the elements denoted with "*" the search gives more then one satisfactory value for $u_L$. Single value is evaluated using the criteria according which, the value of $u_L$ has to be as close as possible to the value of $r_L$. Values in parentheses are approximated values, because the search does not give their exact values. For selection of approximative values minimization of linguistic measure of controller behaviour is used as a criteria.

Table 1. $\Delta y_L$ for $\omega_L$=N

| $u_L$ \ $y_L$ | NB | NM | NS | ZE | PS | PM | PB |
|---|---|---|---|---|---|---|---|
| NB | ZE | NS | NM | NB | NB | NB | NB |
| NM | ZE | NS | NM | NB | NB | NB | NB |
| NS | PS | ZE | NS | NM | NB | NB | NB |
| ZE | PM | PS | ZE | NS | NM | NB | NB |
| PS | PB | PM | PS | ZE | NS | NM | NB |
| PM | PB | PB | PM | PS | ZE | NS | NM |
| PB | PB | PB | PB | PM | PS | ZE | NS |

Table 2. $\Delta y_L$ for $\omega_L$=ZE

| $u_L$ \ $y_L$ | NB | NM | NS | ZE | PS | PM | PB |
|---|---|---|---|---|---|---|---|
| NB | ZE | NS | NM | NB | NB | NB | NB |
| NM | PS | ZE | NS | NM | NB | NB | NB |
| NS | PM | PS | ZE | NS | NM | NB | NB |
| ZE | PB | PM | PS | ZE | NS | NM | NB |
| PS | PB | PB | PM | PS | ZE | NS | NM |
| PM | PB | PB | PB | PM | PS | ZE | NS |
| PB | PB | PB | PB | PB | PM | PS | ZE |

Table 3. $\Delta y_L$ for $\omega_L$=P

| $u_L$ \ $y_L$ | NB | NM | NS | ZE | PS | PM | PB |
|---|---|---|---|---|---|---|---|
| NB | PS | ZE | NS | NM | NB | NB | NB |
| NM | PM | PS | ZE | NS | NM | NB | NB |
| NS | PB | PM | PS | ZE | NS | NM | NB |
| ZE | PB | PB | PM | PS | ZE | NS | NM |
| PS | PB | PB | PB | PM | PS | ZE | NS |
| PM | PB | PB | PB | PB | PM | PS | ZE |
| PB | PB | PB | PB | PB | PM | PS | ZE |

Table 4. $u_L$ for $\omega_L$=N

| $r_L$ \ $y_L$ | NB | NM | NS | ZE | PS | PM | PB |
|---|---|---|---|---|---|---|---|
| NB | NB* | NB* | NB* | NB* | NB* | NB* | NB* |
| NM | NS | NS | NS | NS | NM* | NM* | NM* |
| NS | ZE | ZE | ZE | ZE | ZE | NS* | NS* |
| ZE | PS* | PS | PS | PS | PS | PS | ZE* |
| PS | PS* | PM* | PM | PM | PM | PM | PM |
| PM | PM* | PM* | PB | PB | PB | PB | PB |
| PB | PB* | PB* | PB | (PB) | (PB) | (PB) | (PB) |

Table 5. $u_L$ for $\omega_L$=ZE

| $r_L$ \ $y_L$ | NB | NM | NS | ZE | PS | PM | PB |
|---|---|---|---|---|---|---|---|
| NB | NB | NB | NB | NB | NB | NB | NB |
| NM | NM | NM | NM | NM | NM | NM | NM |
| NS | NS | NS | NS | NS | NS | NS | NS |
| ZE | ZE | ZE | ZE | ZE | ZE | ZE | ZE |
| PS | PS | PS | PS | PS | PS | PS | PS |
| PM | PM | PM | PM | PM | PM | PM | PM |
| PB | PB | PB | PB | PB | PB | PB | PB |

Table 6. $u_L$ for $\omega_L$=P

| $r_L$ \ $y_L$ | NB | NM | NS | ZE | PS | PM | PB |
|---|---|---|---|---|---|---|---|
| NB | (NB) | (NB) | (NB) | (NB) | NB | NB* | NB* |
| NM | NB | NB | NB | NB | NB | NM* | NM* |
| NS | NM | NM | NM | NM | NM | NM* | NS* |
| ZE | ZE* | NS | NS | NS | NS | NS | NS* |
| PS | PS* | PS* | ZE | ZE | ZE | ZE | ZE |
| PM | PM* | PM* | PM* | PS | PS | PS | PS |
| PB | PB* | PB* | PB* | PB* | PB* | PB* | PB* |

The cardinality of the set $R_L$ is 147, and the cardinality of the set $R_L^+$ is 8. The measure of controller incompleteness is $M_I$=0.05.

Let us now define the set of terms $T_{dL}$. Its elements are (in increasing order): very good (VG), good (G), medium (M), bad (B) and very bad (VB). The linguistic function $d_L(A_L, B_L)$ is defined with the Table 7. Approximate values of $u_L$ are estimated by minimization of this function.

Table 7. $d_L(A_L, B_L)$

| $B_L$ \ $A_L$ | NB | NM | NS | ZE | PS | PM | PB |
|---|---|---|---|---|---|---|---|
| NB | VG | G | M | B | VB | VB | VB |
| NM | G | VG | G | M | B | VB | VB |
| NS | M | G | VG | G | M | B | VB |
| ZE | B | M | G | VG | G | M | B |
| PS | VB | B | M | G | VG | G | M |
| PM | VB | VB | B | M | G | VG | G |
| PB | VB | VB | VB | B | M | G | VG |

For example

| $u_L$ | PB | PM | PS | ZE | NS | NM | NB |
|---|---|---|---|---|---|---|---|
| $Q(y_L, r_L, \omega_L)$ | G | M | B | VB | VB | VB | VB |

is the dependance between Q and $u_L$ for $y_L$=ZE, $r_L$=PB and $\omega_L$=N. Q has its minimal linguistic value for $u_L$=PB. This value is used to approximate $u_L$ in controller function $F_C$.

In conclusion it can be pointed out that the measure of behaviour of the designed fuzzy feedforward controller is $\bar{Q}$=G (good).

Fig. 4. shows the response of the process under the control of fuzzy feedforward controller for typical inputs. The results are obtained by simulation.

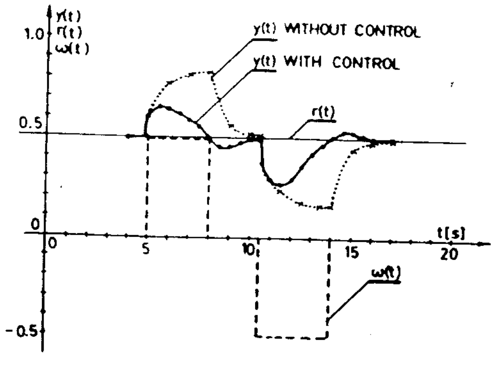


Fig. 4. Response of the process controlled with the fuzzy feedforward controller

## 7. CONCLUSION

Up to present theoretical studies, simulations and practical realisations of fuzzy logic controllers have been aimed to their various applications of feedback control. With the proposed method the area of application of fuzzy logic controller is widened to the feedforward control and the new field of research is opened. In this research the linguistic modelling of the process is specially important, because the linguistic model is considered as a base fo the design of the fuzzy feedforward controller.

The quantitative measure of the controller incompletness and qualitative linguistic measure of the controller behaviour are proposed as the measures of performances of the fuzzy feedforward controller.

Simple example illustrates the proposed method and its effectivness.

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