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FUZZY RELATIONAL MODELS FOR INTELLIGENT CONTROL

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The paper describes how fuzzy set theory could be used in construction of internal models for intelligent control purposes. Fuzzy models and particularly fuzzy relational models are appropriate way for internal model representation of external, surrounding world for systems which interact with rather complex environment. Fuzzy relations are also suitable for representation of control algorithms, so this procedure is described,too.

1. INTRODUCTION

In the last couple of years intelligent robotics has become a field of intensive research, interesting both from theoretical and practical reasons. Such intelligent robotics system is equipped with a number of sensors used to perceive objects in its surrounding. After objects have been perceived the next intelligent robot task is to recognise them and than using this information to plan and execute adequate control action. Typical examples of these actions are to pick an object, to push an object or to move itself in plain or in space avoiding obstacles. In this intelligent robot behavior planning procedure is one of quite important steps, either we are talking about planning of motion of robot hand, for example to pick an object, or we are talking about planning of motion of the robot itself in the case of mobile robot.

The planning procedure is based on the construction of internal robot model of the external, real world. The problem is that complexity of environment in which robot has to act cannot be fully represented in the model. A lot of uncertainty and fuzziness is employed in such robotics system either because of inadequacy of robot receptors and effectors, or because of impossibility to represent objects , to locate objects or to perform actions on objects with sufficient accuracy. Because of all these reasons it is not possible to construct a precise functional mapping between the state-space of the model and the state space of the external world. To solve this problem instead of deterministic approach an alternative nondeterministic one has to be applied.

Among these alternative, one have proved itself on other similar problems. That is an approach based on Zadeh fuzzy set theory. This theory could be used in construction of internal models, as a tool for object representation, for object location description or for explanation of robot action. Examples could be find in [1,2,3].

In this paper we are exploring ideas about using theory of fuzzy sets in construction of internal models for intelligent control purposes. One part of this theory is particularly suitable for this task. That is theory of fuzzy relations and this work is based on it. The starting point was idea of Averkin and Dulin [3] to use fuzzy mapping as a bridge between state-space of internal model and state-space of external world, but in this paper we are going more deeply into this problem.

2. DETERMINISTIC AND NONDETERMINISTIC CONTROL SYSTEM DESCRIPTION

We are analysing control systems of intelligent robots. As any other system this system also has a number of inputs and a number of outputs. Outputs are different control actions as for example control of steering, velocity, acceleration, force etc. Inputs are information received from different sensors usually transformed into form suitable for further processing. Typical example is image information obtained from vision sensors. Input inrmation to control system are results of image analyses: type of objects on the image (results of recognition procedure), object dimensions, object locations and similar.

Generally control system S could be described as a mapping from the set of inputs X and the set of outputs Y.

$$S : X \rightarrow U \qquad (1)$$

It is important to emphasis that this definition does not imply that to each input from X corresponds a unique output from Y. But if this uniqueness is obtained, so mapping (1) is functional and also sets X and Y are subsets of the set of real numbers , than control system S is called deterministic. Sometimes there is a doubt and uncertainty in input and output data and/or mapping (1) is relational. In these cases we are talking about nondeterministic control systems.

Three special cases of nondeterministic control systems could be distinguished, depending is the factor of uncertainty connected with input-output data, with input-output relationships or with both of them.

In the first case uncertainty is connected with representation of external world data and generally in robotics control systems these uncertainty is primarily connected with input, sensory data. For example the distance to an object is not preciously known because of unadequancy of vision and range sensors, so distance information could not be exact real number. Because of that X could not be subset of . This distance information could be a set of real numbers, so X may be defined as a set of intervals of real numbers, or as we will show latter, X could be defined as a set of fuzzy sets defined on .

In the second case uncertainty is connected with control procedures and not with input - output data. Here under term control procedures we mean all procedures connected with control action determination and execution, from interpretation of input data, planning according to this data, evaluating adequate control signals and at the and its execution. Typical example is expert control system which includes knowledge base with knowledge about control in the form of production rules (situation-action pairs), and not by mathematical functions. For formal description of production rules fuzzy relation could be used, too.

The third case is the most general one and in this paper we will concentrate mostly on it.

3. FUZZY RELATIONS

Uncertainty in input-output data and uncertainty in control procedures could be treated using fuzzy relations, so let us first shortly explain what fuzzy relations are.

Fuzzy set is defined as a set to which elements may belong to various degrees, rather than only belong or not belong, as it is the case for classical crisp set. Fuzzy set A* is defined on a crisp set X as a set of ordered pairs A* = = {(x,A(x)}, $x \in X$, $A(x) \in [0,1]$. A(x) is called characteristic function and it express this degree of belongings of element x from X to a fuzzy set A*.

Similarly a binary fuzzy relation R* is relation which may hold between elements of any two crisp sets X and to any degree between 0 and 1. Formally it is a set of ordered pairs R* = = {(x,y),R(x,y)}, where (x,y) is an element of Cartesian product X x Y and R(x,y) is its characteristic function. Generalisation to n-ary fuzzy relation is strength forward.

If X and Y are discrete sets $X=\{x_i\ i \in I\}$ and $Y= = \{y_j\ j \in J\}$ where I and J are index sets, then R* is discrete fuzzy relation and it can be completely given by its fuzzy matrix $\underline{R}$* with components

$$r_{ij} = R(x_i, y_j),\ i \in I,\ j \in J \qquad (2)$$

This fuzzy matrix may be concrete if X and Y are finite sets, otherwise it is only conceptual.

Let us suppose that we have a fuzzy relation R* from X to Y and fuzzy relation P* from Y to Z. Composition o of fuzzy relations R* and P* is also a fuzzy relation S* but from X to Z whose membership function, for each pair (x,z) could be obtained by equation

$$S(x,z) = \sup_{y \in Y} \min (R(x,y),P(y,z)) \qquad (3)$$

In [4] interesting interpretation of this formula is given. S(x,z) could be seen as a strength of a set of chains linking x and z. The strength of such a chain is that of the weakest link, so operation min is performed. But between x and z there are more chains through different y, so the strength of relation between x and z is that of the strongest one (operation supremum over all y from Y).

The composition of finite fuzzy relations can be viewed as a matrix product. With $\underline{R}^* = [r_{ij}]$, $\underline{P}^* = [p_{jk}]$, $\underline{S}^* = [s_{ik}]$ and $\underline{S}^* = \underline{R}^* \circ \underline{P}^*$ we have

$$s_{ik} = \sum_j r_{ij} \otimes p_{jk} \qquad (4)$$

where $\sum$ is in fact operation max and product $\otimes$ operation min. Composition (3) or (4)is usually called sup-min composition.

Let A* and B* be fuzzy sets defined on X and Y respectively. A* implies B* (A* $\rightarrow$ B*) or expressed in words "If A* then B*" is a fuzzy conditional proposition. A mathematical operation for translating this proposition into a fuzzy relation R* in X x Y is called a fuzzy implication operator. There are many possible definitions of this operator, but in control applications usually Mamdani min definition is used:

$$R(x,y) = \min (A(x),B(y)), x\in X, y\in Y \quad (5)$$

Union and intersection of fuzzy sets, linguistically expressed as connectives 'or' and 'and' are usually defined with operations max and min respectively.

These are just few definitions from the fuzzy set theory important for fuzzy relational model construction.

4. FUZZY RELATIONAL MODELS

Fuzzy relational models are appropriate way to represent uncertainty of the external world. They can be used in cases when it is not possible to construct a precise functional mapping between the state-space of internal model and state-space of external world. Values of membership functions of fuzzy modelling relations could be seen as degrees of similarity between the world and the model or as degrees of precision of the real world description.

Fuzzy modelling relation R* is a binary fuzzy relation between the world state-space W and the model state-space M. For example R* could be seen as a fuzzy matrix whose columns correspond to robot's world state-space, let us say to discrete values of passaway width through which the mobile robot must pass. Rows of fuzzy matrix R* correspond to robot's internal, model state-space, for example to elements symbolically expressed with words of natural language:'very wide'(VW), 'wide'(W), 'narrow'(N), etc. Table 1. is typical simple example. Elements of this table express degrees to which elements of the world state-space W belong to elements of the model state-space M and vice versa.

Table 1. Fuzzy modelling relation for passaway width -RI* (values of W are in m)

M↓ W→	0.5	1	1.5	2
very width(VW)	0	0.1	0.25	1
wide (W)	0	0.3	0.5	1
narrow (N)	1	0.7	0	0

Each row of Table 1. defines a membership function of fuzzy set m_i^* from the model state-space whose support set is the real world state-space W. Situation is similar for each column of Table 1., which defines a membership function of fuzzy set w_j^* from the world state-space whose support set is state-space of the model M. Typical examples is fuzzy set 'wide' whose membership function is fuzzy vector [0 0.3 0.5 1]. This fuzzy vector says that, for example real value 1.5 belongs to concept expressed with 'wide' with degree 0.5. Also real value 1.5 m could be seen as fuzzy set '1.5 m' defined on the model state-space with fuzzy vector [0.25 0.5 0]. This means that for example symbolic value very wide belongs to fuzzy set '1.5 m' with degree 0.25. Elements of the real world are not fuzzy in the real world state-space, but they become fuzzy in the model world state-space and vice versa.

Important is that using this approach it is possible to construct internal model representation of the real world with various levels of abstraction. The level of abstraction is directly connected with cardinallity of the model state-space M. At the lowest level of abstraction the state-space of the model is the same as the state space of the world (M = W). Special case is non-fuzzy, functional case when modelling relation is Boolean and one-to-one. The real abstraction begin when state-spaces M and W are not any more sets with the same elements. Special case with Boolean, but not one-to-one mapping is quantisation of real line For example an element symbolically expressed with '0.5' or I or #1 may stay for all real values between 0 and 0.5.

If we introduce more elements in the model state-space W, for example terms 'very very wide' or 'not so wide' than level of abstraction diminish and contrary reducing the cardinallity of the set W the level of abstraction increase. Model state-space with only two elements 'wide' and 'narrow' is more abstract than one given on Table 1.

Important is to notice that in the case when the model state-space M has more elements than the world state-space W we have situation opposite to abstraction, we have some kind of interpolation.

Let us now suppose that we have fuzzy modelling relation for both inputs and outputs of robot control system. Table 1. could be example of fuzzy relational model for input information "passaway width" and Table 2. for output information "robot velocity".

Table 2. Fuzzy modelling relation for robot velocity-RO*(value of W are in m/s)

M↓ W→	0.2	0.5	0.8	1	1.5
high (H)	0	0	0.2	0.8	1
medium (M)	0.2	0.7	1	0.8	0.1
low (L)	1	0.9	0.2	0	0

The task of the control system is to plan the robot velocity according to specific input information about passaway width. Let us suppose that velocity could be really adjusted only in discrete steps from Table 2. and that information about passaway width are also discrete one from Table 1.

The final result of the planning procedure, which must be input information to low level control, must be one and only one element of the robot velocity world state-space. This means that finally each controller, which act in real world must be deterministic controller. But on the model level it is not necessary to have deterministic procedure. Moreover non-deterministic, fuzzy procedure is more close to description of control procedure which humans use during control. Let us suppose that we are using human knowledge for our passaway width - robot velocity planning task. The planning procedure could be expressed with production rules, for example:

"If passaway width is very wide,
than velocity could be high, or
if passaway width is wide,
than velocity could be high, too, or
if passaway width is narrow,
than velocity could be low" ....(6)

Conventional procedure is to express these rules with fuzzy conditional propositions connected with union ($\underline{VW}* \rightarrow \underline{H}* \cup \underline{H}* \rightarrow \underline{H}* \cup \underline{N}* \rightarrow \underline{L}*$) where $\underline{VW}*$, $\underline{H}*$, $\underline{W}*$, $\underline{N}*$ and $\underline{L}*$ are fuzzy vectors obtained as rows of Table 1. and 2. To transform these rules in the fuzzy relation between the real world state-space of passway width and the real world state-space of robot velocity, fuzzy implication operator must be used. For example if we define fuzzy implication with (5) and union with max the final result is Table 3.

Table 3. Fuzzy modelling relation of control algorithm

WI↓ WO→	0.2	0.5	0.8	1	1.5
0.5	1	0.9	0.2	0	0
1	0.7	0.7	0.2	0.3	0.3
1.5	0	0	0.2	0.5	0.5
2	0	0	0.2	0.8	1

The final step is interpretation. This means changing Table 3. into Boolean table which will have in each row one and only one non-zero element and it must be equal to 1.

Let us now show how this procedure could be seen from the position of theory of fuzzy relations. Production rules could be seen as a relation between the model state-space of passaway width and the model state-space of the robot velocity. For production rules of the form (6) this relation is Boolean and can be expressed with Table 4.

Table 4. Production rules as relation-RP*

MI↓ MO→	high	medium	low
very wide	1	0	0
wide	1	0	0
narrow	0	0	1

As elements of fuzzy modelling relation of control algorithm could be seen as degrees of strength between input and output world state-space, natural way to obtain this relation is by composition of fuzzy relations RI*, RP* and RO* (Boolean relation RP* from Table 4. is also only a special case of fuzzy relation):

$$RC* = RI* \circ RP* \circ RO* \qquad (7)$$

Interesting is that results obtained with this equation, where composition o is given with (3), or more preciously with (4), completely coincide with Table 3. obtained by fuzzy implication operator (5) and definition of union with max. This equality could be proved, so maybe that is the reason why Mamdani min definition of fuzzy implication operator is the most appropriate one for control.

This approach which use composition of relations, instead of fuzzy implications has one additional advantage, one additional degree of freedom. That is the possibility to use, in representation of production rules, instead of Boolean relation a real fuzzy relation. This means the use production rules of the form:

"If passaway width is very wide, than velocity could be high with degree 1, medium with degree 0.8 or law with degree 0.2", or

"The velocity could be high, if passaway width is very wide with degree 1, wide with degree 0.9 or narrow with degree 0.1"

Existing fuzzy control algorithms are mostly based on fuzzy implications which led to Boolean relations. This fuzzy approach is a novelty and it can be used in cases when it is not possible to obtain consensus about control actions for certain inputs.

5. FUZZY RELATIONAL MODELS IN MOTION PLANNING AND CONTROL OF MOBILE ROBOTS

Practical application of these ideas about fuzzy modelling relations could be motion planning and control of mobile robot or autonomous vehicle. An example is unmanned submersible which must act in underwater surroundings which is not very well known.

Typical decomposition of robot's tasks are planning, navigation and piloting [5]. For each of these tasks fuzzy modelling relation with different levels of abstraction could be used. At the highest, but least precise and detailed level is the *planner* which operate on incomplete, global map to determine connected sub goals for specific tasks. The level of abstraction is here the highest one. Real world is described just roughly with approximate model. The next level is *navigator* which utilities a more detailed map to evaluate an obstacle free local path which will satisfies some performance criteria. With this path description the *pilot* provides motion control avoiding obstacles not identified by navigator. It needs the most precise information and the less abstract model state-space.

This operation of decreasing the level of abstraction of internal model state-space could be called *zooming of abstraction*. It can be defined as a procedure which increase the cardinallity of the model state-space. Complementary to this could be *zooming of fuzziness* when values of fuzzy relation membership function are changed such that fuzziness is diminished (values are more closed to 1 and 0). The third procedure could be *zooming of precision* defined as a procedure which increase the cardinallity of the world state-space. For example we change the world state-space of the passaway width from Table 1. such that it has not any more only five elements for passawsy widths between 0 and 2 m, but let us say ten elements.

This three procedures could be used together to obtain more or less precise picture of external, real world for different tasks of mobile robot motion planning and control.

6 CONCLUSION

Fuzzy models and particularly fuzzy relational models could be used as an adequate way for internal, model representation of not well known, or not precisely known eternal world. They could be used as a bridge between rough, approximate, symbolic state-space of the model and more precise, numeric state-space of the world.

Using this approach it is possible to represent the same situation with hierarchically organised fuzzy models whose degree of abstraction increase or decrease. Typical field of application of this approach could be motion planning and control of mobile robot or autonomous vehicle.

Fuzzy relational models could be also used for representation of control rules. Approach described in this paper use composition of fuzzy relations to obtain control fuzzy relation. This approach is suitable for cases when it is not possible to obtain consensus about control algorithm expressed with production rules.

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REFERENCES

[1] Doods,D.R., Fuzziness in knowledge based robotic systems, *Fuzzy Sets and Systems*,26 (1988),179-193

[2] Lea,R.N. and Johnson,L.B., Fuzzy sets and autonomous navigation, *SPIE Proc.Conf.on Appl. of AI*, 786, (1987), 448-452

[3] Averkin,A.N. and Dulin,S.K., Fuzzy modelling relations for robots, *Proc.of IFAC Symp. on AI*, (1983), 331-336

[4] Dubois,D. and Prade,H., *Fuzzy Sets and Systems: Theory and Application*, Academic Press, New York,1980., p.74

[5] Isik,C. and Meyers,A.M., Pilot level of a hierarchical controller for an unmanned mobile robot, *IEEE Journal of Robotics and Automation*, 4(3), (1988), 241-255