

MODELS OF NONDETERMINISTIC SYSTEMS FOR  
 APPLICATION IN FEEDFORWARD CONTROL

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ABSTRACT

The paper presents fuzzy models of ill-defined, nondeterministic systems suitable for application in feedforward control.

Formal and linguistic approach to fuzzy model evaluation are distinguished, and several forms of static and dynamic fuzzy models suitable for practical applications are discussed.

Fuzzy feedforward control algorithms for both approaches are developed, based on the theory of fuzzy relational equations and theory of linguistic modelling and approximative reasoning.

Simple numerical example illustrates the proposed methods and its effectiveness.

This fuzzy approach to modelling of nondeterministic systems and to feedforward control is a complementary approach to conventional, deterministic or stochastic one: It enables to apply the principles of invariant automatic control to situations where conventional methods do not give desired quality.

INTRODUCTION

Invariant control may be defined as a control policy by which the total or partial independence of controlled system with respect to disturbances acting upon it has to be achieved. It has been proved that the feedforward control is much more convenient as the invariant control, then the feedback one, especially when a high degree of system invariance to disturbances has to be achieved as a control goal [1].

For the successful application of feedforward control the model which describes the primarily effects of disturbance and manipulated inputs on the system behaviour has to be known. During last two decades the feedforward control have been studied and applied in numerous systems on the basis of their deterministic models.

But there are many situations in which our understanding of system behaviour is imprecise and inadequate. For a great number of systems, for example various industrial processes, and especially systems in "soft" sciences, their complexity, limited knowledge, uncertainty or stochastic environment prevent us to give unique and reliable formulation of input-output relationship, and develop proper and accurate deterministic models. Partly that is a result of the "Principle of incompatibility" introduced by Zadeh [2] which states that as system becomes more complex, it becomes

increasingly difficult to make statements about them which are both meaningful and precise. If we want to apply invariant, feedforward control also in these cases, first we have to develop adequate models of such nondeterministic systems.

Distinguishing the spaces of disturbance input,  $Z_D$ , manipulated input,  $Z_M$ , and controlled output,  $Y$ , the general model of the system can be given as a complex

$$M = \{Z_D, Z_M, Y, R\} \quad (1)$$

where for the dynamic discrete model  $R$  is a relation between the sequence of the controlled output, manipulated input and disturbance input defined in adequate discrete time moments. For the  $k$ -th order system  $R$  is a relation of the  $k+2$  order defined on the Cartesian product

$$Z_D \times Z_M \times \underbrace{Y \times \dots \times Y}_{k\text{-times}}$$

If the values of the input and output variables are single elements belonging to  $Z_D, Z_M$  and  $Y$ , and  $R$  has the property

$$\forall \begin{matrix} z_D \in Z_D \\ z_M \in Z_M \\ y_1, \dots, y_{k-1} \in Y \end{matrix} \exists! R(z_D, z_M, y_1, \dots, y_k) = \begin{cases} 1, & \text{if } y_k = Y \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

$M$  represents the deterministic system, otherwise  $M$  represents the nondeterministic system [3].

In recent years the use of fuzzy set theory [4] has proved a favorable approach to modelling such nondeterministic systems. If the factor of imprecision and uncertainty appears in the expression of input and output data, their values can be defined as fuzzy sets. If the factor of imprecision and uncertainty is connected with input-output relationships,  $R$  can be defined as a fuzzy relation. In any one of these cases model  $M$  is called fuzzy model of nondeterministic system.

The fuzzy set theory also gives the methodology for the development of the control algorithms and their implementation on computer control systems [5]. In the rest of this paper we will deal with the new methods of fuzzy modelling of nondeterministic systems suitable for feedforward control and their practical application in real-time control procedures. But before the presentation of our ideas and methods brief survey of basic knowledge necessary to describe fuzzy models is given.

Let  $W$  be an ordinary set, and  $\mu_A$  a fun-

ction which assigns to each element  $w$  in  $W$  a number in the interval  $[0,1]$ . The function  $\mu_{A^*}$  specifies a fuzzy set  $A^*$  of the set  $W$ .  $\mu_{A^*}$  is called membership function of the fuzzy set  $A^*$ , and  $W$  universe of discourse.

Let  $W_1, \dots, W_k$  are ordinary sets. The function,  $\mu_{R^*}$ , which assigns to each element  $(w_1, \dots, w_k)$  in Cartesian product  $W_1 \times \dots \times W_k$  a number in interval  $[0,1]$  specifies a fuzzy relation  $R^*$  of the set  $W_1 \times \dots \times W_k$ .

The union ( $\cup$ ) and intersection ( $\cap$ ) of fuzzy sets  $A^*$  and  $B^*$  from  $W$  are defined as

$$C^* = A^* \cup B^* \Leftrightarrow \mu_{C^*}(w) = \max(\mu_{A^*}(w), \mu_{B^*}(w)) \quad (3)$$

$$D^* = A^* \cap B^* \Leftrightarrow \mu_{D^*}(w) = \min(\mu_{A^*}(w), \mu_{B^*}(w)) \quad (4)$$

Max-min ( $\circ$ ) and  $\alpha$ -composition ( $\otimes$ ) of the fuzzy set  $A^*$  and fuzzy relation  $R^*$  are defined as

$$B^* = R^* \circ A^* \Leftrightarrow \mu_{B^*}(w_2) = \sup_{w_1 \in W_1} (\min(\mu_{R^*}(w_1, w_2), \mu_{A^*}(w_1))) \quad (5)$$

$$C^* = R^* \otimes A^* \Leftrightarrow \mu_{C^*}(w_2) = \inf_{w_1 \in W_1} (\mu_{R^*}(w_1, w_2) \alpha \mu_{A^*}(w_1)) \quad (6)$$

where

$$\mu_{R^*}(w_1, w_2) \alpha \mu_{A^*}(w_1) = \begin{cases} 1 & , \text{ IF } \mu_{R^*}(w_1, w_2) \leq \mu_{A^*}(w_1) \\ \mu_{A^*}(w_1) & , \text{ IF } \mu_{R^*}(w_1, w_2) > \mu_{A^*}(w_1) \end{cases} \quad (7)$$

The Cartesian product of fuzzy sets  $A^*$  from  $W_1$  and  $B^*$  from  $W_2$  is defined as

$$C^* = A^* \times B^* \Leftrightarrow \mu_{C^*}(w_1, w_2) = \min(\mu_{A^*}(w_1), \mu_{B^*}(w_2)) \quad (8)$$

The fuzzy discretisation [6] is a method which make possible the use of fuzzy and non-fuzzy forms of information. It consists of cutting up a real space  $W$  into a series of non-disconnected fuzzy sets  $Z_1^*, Z_2^*, \dots, Z_I^*$  such that the entire space  $W$  is covered:

$$\forall w \in W \quad \exists 1 \leq i \leq I : \mu_{Z_i^*}(w) > 0 \quad (9)$$

Thus every fuzzy and nonfuzzy set which belongs to space  $W$  can be represented by fuzzy vectors in terms of  $Z_i^*$ .

For example, real value  $w_0 \in W$  can be represented by

$$\underline{Z}^* = [\mu_{Z_1^*}(w_0) \quad \dots \quad \mu_{Z_I^*}(w_0)] \quad (10)$$

and fuzzy set  $A^* \subset W$  by

$$\underline{Z}^* = [\sup_{w \in W} (\min(\mu_{Z_1^*}(w), \mu_{A^*}(w))) \quad \dots \quad \sup_{w \in W} (\min(\mu_{Z_I^*}(w), \mu_{A^*}(w)))] \quad (11)$$

## FUZZY MODELS OF NONDETERMINISTIC SYSTEMS

The problem of the fuzzy models evaluation is a part of the theory of fuzzy systems, and the reasearch on fuzzy systems have developed in two main directions [7]. The first one is a formal approach and it considers fuzzy systems as a generalization of nondeterministic systems. The second direction is a linguistic approach in which a fuzzy model is viewed as a mathematical meaning of the linguistic description of the system behaviour. Following these directions we apply both approaches for the evaluation of fuzzy models suitable for application in feedforward control

## Formal approach

Generally the fuzzy model of the system suitable for feedforward control can be defined as a complex

$$M = \{Z_D, Z_M, Y, R^*\} \quad (12)$$

where  $Z_D, Z_M$  and  $Y$  are real spaces defined in (1) and  $R^*$  is a fuzzy relation which describes the relationship between system input and output variables.

If in the system description the principle of superposition can be applied, the fuzzy model (12) can be changed to a form

$$M = \{Z_D, Z_M, Y, R_D^*, R_M^*\} \quad (13)$$

where  $R_D^*$  and  $R_M^*$  are fuzzy relations describing the relationship between the disturbance input and controlled output and the relationship between the manipulated input and controlled output, in both cases with another input equals to zero.

Static and dynamic fuzzy models can be distinguished, and the difference is only in fuzzy relations  $R^*$ ,  $R_D^*$  and  $R_M^*$ . In static fuzzy models the fuzzy relations describe the relationship between inputs and outputs in steady-state conditions, for example the fuzzy relation  $R^*$  of the nonlinear system with one disturbance and one manipulated input and one output is  $R^*(z_D, z_M, y, y_{SS})$ , where index SS means that the values of output variable are considered in steady-state condition. This fuzzy relation is defined on Cartesian product  $Z_D \times Z_M \times Y \times Y$ . For practical applications it is more convenient to use system output increment,  $\Delta y_{SS}$ , as a system output variable instead of the controlled output only,  $y_{SS}$ . Also it is not necessary to considered the values of the output variable in steady state condition. The developed fuzzy feedforward control algorithms work quite satisfactory if the static fuzzy model is evaluated using the values of the output variable some discrete time interval after the inputs are applied. To this interval the system time delays has to be added. If we may consider that the system is linear, the fuzzy relations are more simple because the value of actual output has not to be incorporate. For example the fuzzy relations  $R_D^*$  and  $R_M^*$  of the system with linear behaviour with one of each inputs and one output are  $R_D^*(z_D, \Delta y_{SS})$  and  $R_M^*(z_M, \Delta y_{SS})$ .

In dynamic fuzzy models only discrete models will be considered, because of the inadequacy of fuzzy integration methods. The fuzzy relation of the  $k$ -th order system describes the relationship between a sequence of controlled output in discrete time moment  $nT, (n+1)T, \dots, (n+k-1)T$ , manipulated input applied in the moment  $((n+k)T - t_{dM})$ , disturbance input applied in the moment  $((n+k)T - t_{dD})$  and controlled output or controlled output increment in discrete time moment  $(n+k)T$ .  $t_{dM}$  and  $t_{dD}$  are delays of the manipulated input-controlled output and disturbance input-controlled output paths of the system. For the sake of simplicity we shall concentrate on the first order systems, with one of each inputs and one output. In this case the fuzzy relations are  $R^*(z_D, z_M, y, \Delta y)$ ,  $R_D^*(z_D, y, \Delta y)$  and  $R_M^*(z_M, y, \Delta y)$ .

System variables usually have nominal operating values. Considering system behaviour in vicinity of these values instead of input variables,  $z_D$  and  $z_M$ , their increment can be used,  $\Delta z_D$  and  $\Delta z_M$ .

Using fuzzy relations, the system behaviour can be simulated by fuzzy difference relational equations, for example

$$\Delta y_n^* = z_{Dn}^* \circ z_{Mn}^* \circ y_n^* \circ R^* \quad (14)$$

where  $\Delta y_n^* \in \Delta Y$ ,  $z_{Dn}^* \in Z_D$ ,  $z_{Mn}^* \in Z_M$ ,  $y_n^* \in Y$  are appropriate fuzzy sets in the moment  $nT$ , and  $\circ$  is relation-relation composition operator. We have used max-min composition and substitute  $\circ$  with  $\circ$ .

There are two main problems in fuzzy models evaluation. The first one is problem of collection of input-output data, and the second one is the problem of identification of fuzzy relations from this data.

Two procedures of the collection of input-output data can be distinguished. In the first procedure the experience and knowledge of human operators and/or experts is used. From their approximative description of process behaviour, sometimes, like in linguistic approach with linguistic causal statements, the set of input-output fuzzy sets are obtained. This method will be explain in more details in the next chapter. Then using fuzzy discretisation method the fuzzy vectors are calculated with equation (11), and the final results is the set of input-output fuzzy vectors. In the second procedure the real data, as a result of direct observation of system behaviour, are used. One of the reason why the fuzzy model is constructed from this data and not the deterministic one is for example the data are imprecise because of inaccurate and inadequate measuring equipment. By the equation (10) appropriate fuzzy vectors are calculated and the final results is the same as in the first procedure: the set of input-output fuzzy vectors. Because of that the combination of both approaches can be used.

Now the identification problem can be stated as follows:

Define the fuzzy matrix  $R^*$  (or matrices  $R_D^*$  and  $R_M^*$ ) that satisfies the fuzzy relational equations for input-output fuzzy vectors obtained in collection procedure. For example if the set of fuzzy vectors is  $\{(z_{D1}^*, z_{M1}^*, y_1^*, \Delta y_1^*)\}$   $i=1, \dots, I$ , the fuzzy relational equations which have to be satisfied are

$$\Delta y_i^* = z_{Di}^* \circ z_{Mi}^* \circ y_i^* \circ R^*, i=1, \dots, I \quad (15)$$

The simplest and ideal situation is when each of  $(z_{Di}^*, z_{Mi}^*, y_i^*, \Delta y_i^*)$  satisfies the fuzzy equation (15). According to Sanches [8] the greatest fuzzy relation  $\hat{R}^*$ , such that fuzzy equations (15) are valid is

$$\hat{R}^* = \bigcap_{i=1}^I ((z_{Di}^* \circ z_{Mi}^* \circ y_i^*) @ \Delta y_i^*) \quad (16)$$

In practical situation the assumption that each fuzzy equation is satisfied usually is not valid, and then the methods of the identification of fuzzy systems [3,9] have to be used.

The final result of the formal approach is the model of nondeterministic system given in the form of fuzzy relation (or relations). This fuzzy model is essential part of fuzzy feedforward control algorithms.

#### Linguistic approach

In linguistic approach the verbal model of nondeterministic system has to be developed first. Generally the verbal model can be defined as a complex

$$M^L = \{T(z_D^L), T(z_M^L), T(y^L), R^L\} \quad (17)$$

where  $T(z_D^L)$ ,  $T(z_M^L)$ ,  $T(y^L)$  are sets of linguistic values of linguistic variables of disturbance input,  $z_D^L$ , manipulated input,  $z_M^L$  and controlled output,  $y^L$ , usually called sets of terms, and  $R^L$  is a set of causal statements which describes linguistically the dependence of system input and output variables.

Like in formal approach the static and dynamic verbal models can be distinguished, and for all specific situations (linear, nonlinear systems, systems where principle of superposition holds, systems with nominal values of input variables) adequate verbal models can be defined. An example of a causal statement of static verbal model is:

- If  $z_D^L$  is "small" and  $z_M^L$  is "big" and  $y^L$  is "medium", then the output in steady-state condition  $y_{ss}^L$  will be "between big and very big" or of dynamic verbal model:

- If in the moment  $nT$   $z_{Dn}^L$  is "moderate bellow nominal value" and  $z_{Mn}^L$  is "at nominal value" and  $y_n^L$  is "at nominal value", then the output increment  $\Delta y_n^L$  will be "negative small".

The set of causal statements  $R^L$  can be obtained by various methods [10], using the knowledge of experts who knows the system behaviour.

Symbolically the causal statement can be expressed with linguistic implication, for example

$$r^L : (a_{z_{Di}}^L, a_{z_{Mi}}^L, a_{y_i}^L) \rightarrow a_{\Delta y_i}^L \quad (18)$$

where  $a_{z_{Di}}^L \in T(z_D^L)$ ,  $a_{z_{Mi}}^L \in T(z_M^L)$ ,  $a_{y_i}^L \in T(y^L)$  and  $a_{\Delta y_i}^L \in T(\Delta y^L)$ .

The statements are connected together with the word "also". Defining the fuzzy language [11] to each linguistic value adequate fuzzy set is assigned. These fuzzy sets give mathematical meaning to linguistic values. The mathematical meaning to the set of causal statements  $R^L$  gives the fuzzy relation  $R^*$ . For statements expressed with (18)  $R^*$  can be defined with

$$R^* = f_{\text{also}}(f_{\rightarrow}(f_{\text{and}}(\mu_{a_{z_{Di}}}^L(z_D), \mu_{a_{z_{Mi}}}^L(z_M), \mu_{a_{y_i}}^L(y)), \mu_{a_{\Delta y_i}}^L(\Delta y))) \quad (19)$$

where  $f_{\text{also}}$ ,  $f_{\text{and}}$  and  $f_{\rightarrow}$  are operations of fuzzy sets which give meaning to the words "also", "and", and to the linguistic implication (conjunction "if...then").

Various operations for  $f_{\text{also}}$ ,  $f_{\text{and}}$  and especially  $f_{\rightarrow}$  have been proposed. In our work we define  $f_{\text{also}}$  as a union of fuzzy sets,  $f_{\text{and}}$  as an intersection, and  $f_{\rightarrow}$  as a Cartesian product. The reason for this is because we apply max-min composition, and it is unreasonable to infer anything at all if antecedent in linguistic implication is not true. In that cases another statements gives the answer what will be as a consequent.

So we can conclude that the final results of linguistic approach is also the fuzzy relation (or relations) which contain the information of the behaviour of nondeterministic systems, and it is (or they are) essential part of fuzzy feedforward control algorithms, too.

#### FUZZY FEEDFORWARD CONTROL ALGORITHMS

The fuzzy feedforward control action is formulated by means of two off-line and on-line procedure. The off-line procedures are:

- the determination of the control action sign, and
- the determination of the control action delay, and the on-line procedure is
- the determination of the control action magnitude.

If the trends of the controlled output changes are the same for the same trends of input variable changes, the control action sign will be minus, and vice versa.

The values of the control action delay is determined as the difference between the delay of the disturbance input-controlled output,  $t_{dD}$ , and the delay of the manipulate input-controlled output,  $t_{dM}$ , paths of the system. The controller will act successfully, as all others feedforward controllers do, only if  $t_{dD} \geq t_{dM}$ .

In the procedure for determination of the control action magnitude, formal and linguistic approach can be distinguished. Normally in each of these approaches the fuzzy relations obtained with the same approach are used.

Let us start with formal approach, and suppose for example that the fuzzy matrix, of dynamic fuzzy model has the form  $R^*(\Delta z_D, \Delta z_M, y, \Delta y)$ . In the time instant  $nT$  the input information are fuzzy vectors of disturbance input  $\Delta z_{Dn}^*$  and actual output  $y_n^*$  obtained with the equation (10) or (11). The control action magnitude is calculated by one of the interpretation procedures [6] (for example mean value method) from the fuzzy vector  $\Delta z_{Mn}^*$  obtained as a solution of the equation

$$\Delta y_n^* = \Delta z_{Mn}^* \circ \Delta z_{Dn}^* \circ y_n^* \circ R^* \quad (20)$$

where  $\Delta y_n^*$  corresponds to the real value 0, or linguistic value "zero".

If the solution exist the least upper bound  $\hat{\Delta z}_{Mn}^*$  of all solutions can be calculated by the equation [7]:

$$\hat{\Delta z}_{Mn}^* = (\Delta z_{Dn}^* \circ y_n^* \circ R^*) @ \Delta y_n^* \quad (21)$$

If the exact solution does not exist various methods for the calculation of approximate solution can be used [3,12].

In linguistic approach generalized modus tollens [7] and compositional rule of inference [2] are applied. For example for the fuzzy matrix of the same form as in formal approach, but obtained with linguistic identification procedure, the fuzzy vector of the manipulated input increment is calculated with the equation

reference [2] are applied. For example for the fuzzy matrix of the same form as in formal approach, but obtained with linguistic identification procedure, the fuzzy vector of the manipulated input increment is calculated with the equation

$$\Delta z_{Mn}^* = \Delta y_n^* \circ \Delta z_{Dn}^* \circ y_n^* \circ R^* \quad (22)$$

The real control action magnitude can be obtained by any one of the interpretation methods.

The same procedure can be easily extended to all other forms of fuzzy models.

#### NUMERICAL EXAMPLE

The presented methods are illustrated by a simple numerical example.

Fig. 1. illustrates a hot-water heater that mixes cold water with steam to produce hot water. The hot water temperature is controlled output, steam flow manipulated input and cold water flow disturbance input. The deterministic representation of considered process is also given, because the process was simulated on analog computer.

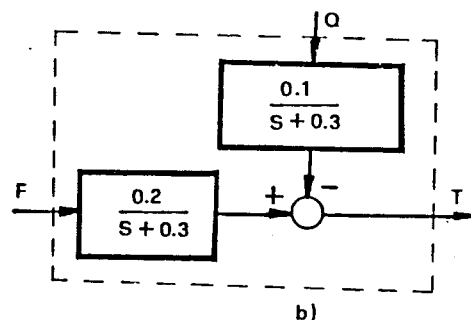
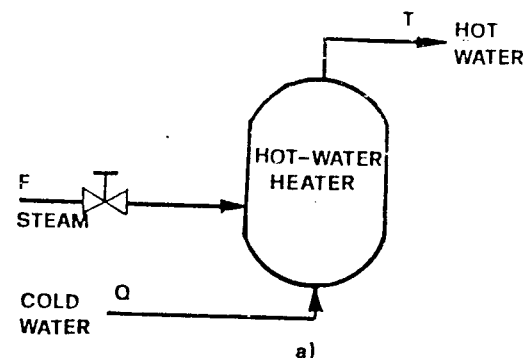


Fig.1.a) The hot-water heater

b) The deterministic represent of the considered process (time constants in minutes)

For convenience let us suppose that nominal operating values of the process variables are  $F_N=10, Q_N=T_N=5$ , and that instead of deterministic model only the set of cause-effect numeric data are known:

$$S_D = \{(\Delta Q, \Delta T)\} = \{(-4.5, 0.18), (-2.1, 0.051), (-0.9, 0.015), (0, 0), (1.3, -0.05), (3, -0.14), (5, -0.21)\}$$

$$S_M = \{(\Delta F, \Delta T) = (-4.5, -0.42), (-2, -0.22), (-1, -0.15), (0, 0), (0.5, 0.1), (3, 0.28), (4.7, 0.43)\}$$

Each pair can be linguistically interpreted with conditional statement, for example statement for pair (-4.5, 0.18) is: "If cold water flow decrease 4.5 units below nominal value, then the hot water temperature will increase 0.18 units 0.5 minutes later".

Defining basic fuzzy sets with Fig.2, and using equation (10) pairs  $(\Delta Q, \Delta T)$  and  $(\Delta F, \Delta T)$  can be transformed in pairs of fuzzy vectors  $(\underline{\Delta Q}^*, \underline{\Delta T}^*)$  and  $(\underline{\Delta F}^*, \underline{\Delta T}^*)$ .

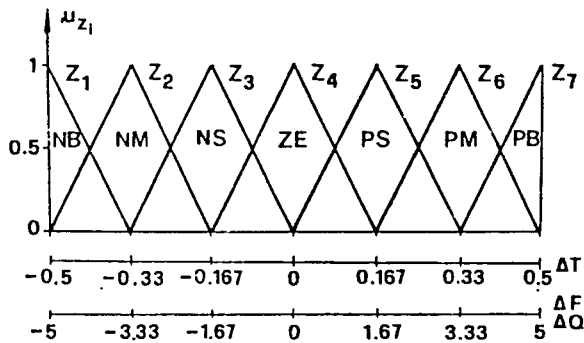


Fig.2. Basic fuzzy sets of fuzzy discretisation with their linguistic labels

On Fig.2. linguistic labels of basic fuzzy sets are given, too. NB means negative big, NM-negative medium, NS-negative small, ZE-zero, and the same for positive values.

Using either formal or linguistic approach fuzzy matrices whose form is  $R_D^*(\Delta Q, \Delta T)$  and  $R_M^*(\Delta F, \Delta T)$  can be calculated. Although each pair does not satisfy its appropriate fuzzy relational equation

$$\underline{\Delta T}^* = \underline{\Delta Q}^* \circ R_D^* \quad (23)$$

$$\underline{\Delta T}^* = \underline{\Delta F}^* \circ R_M^* \quad (24)$$

in this example in formal approach equation (16) was used for fuzzy matrices evaluation. In linguistic approach  $R_D^*$  and  $R_M^*$  were calculated with equation (19), defining  $f_{\cup}$  also with union,  $f_{\cap}$  and with intersection and  $f_{\rightarrow}$  with Cartesian product of fuzzy sets.

Fuzzy matrices  $R_D^*$  and  $R_M^*$  obtained with formal approach are

$$R_D^* = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.204 \\ 0 & 0 & 0 & 0 & 0.3 & 1 & 0.796 \\ 0 & 0 & 0.695 & 1 & 0.162 & 0.162 & 0 \\ 1 & 1 & 0.09 & 0 & 0 & 0 & 0 \\ 0.078 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_M^* = \begin{bmatrix} 0.52 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.48 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.68 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.68 & 0.42 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.58 \end{bmatrix}$$

and with linguistic approach

$$R_D^* = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.204 \\ 0 & 0 & 0 & 0.222 & 0.3 & 0.802 & 0.796 \\ 0 & 0.263 & 0.695 & 1 & 0.7 & 0.162 & 0 \\ 0.7 & 0.3 & 0.305 & 0.09 & 0 & 0 & 0 \\ 0.078 & 0.078 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_M^* = \begin{bmatrix} 0.56 & 0.3 & 0 & 0 & 0 & 0 & 0 \\ 0.48 & 0.3 & 0.32 & 0 & 0 & 0 & 0 \\ 0 & 0.198 & 0.68 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 1 & 0.3 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & 0.3 & 0.32 & 0 \\ 0 & 0 & 0 & 0 & 0.198 & 0.68 & 0.42 \\ 0 & 0 & 0 & 0 & 0 & 0.18 & 0.58 \end{bmatrix}$$

Fuzzy model can be easily converted into a linguistic model, in which each statement has its degree of possibility. Some of the statements for the formal approach are as follows:

"possibility that  $\Delta T$  is NS if  $\Delta Q$  is PB is 0.796"

"possibility that  $\Delta T$  is PM if  $\Delta F$  is PB is 0.42"

Fig.3. shows the fuzzy feedforward control loop.

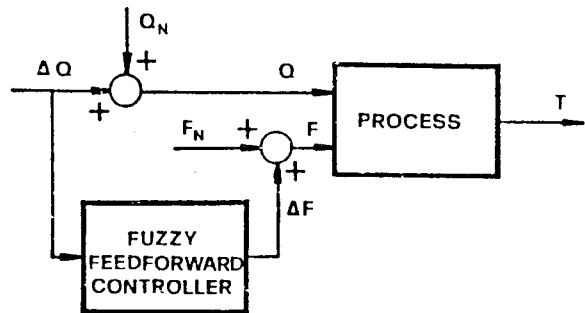


Fig.3. Fuzzy feedforward control loop

Control algorithms were applied on small microcomputer control system, and process was simulated on analog computer.

Fuzzy feedforward control actions obtained with formal and linguistic approaches are compared in Fig.4. Disturbance input, and process response without control are given, too.

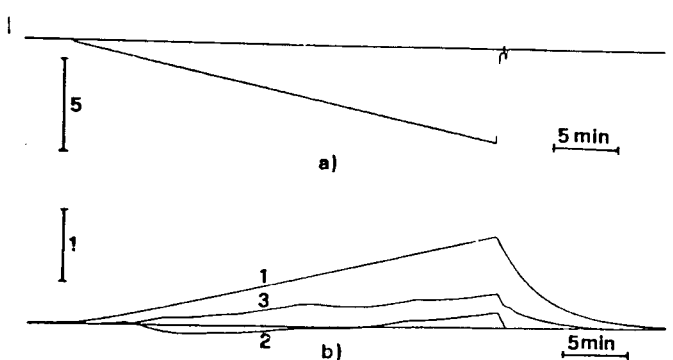


Fig.4. a) Disturbance input; b) Process response without control (1) and with fuzzy feedforward control for formal (2) and linguistic approach (3)

In order to compare the fuzzy approach with conventional approach a linear regression model is considered. For the same input-output data the regression model present  $\Delta T = -0.0403 \cdot \Delta Q$  and  $\Delta T = 0.0953 \cdot \Delta P$ .

Fig.5. shows the process response for conventional and fuzzy feedforward control (formal approach) for pulse disturbance and saw tooth disturbance (the same disturbance as in Fig.4a)

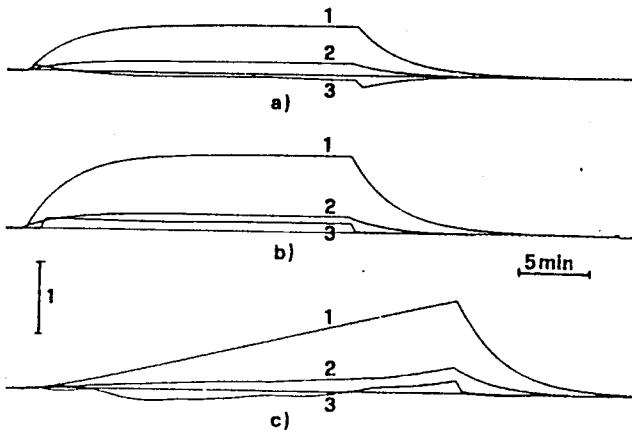


Fig.5. Process response without control (1), with conventional feedforward control (2) and fuzzy feedforward control-formal approach (3) for  
a) pulse disturbance with amplitude -2.5 units  
b) pulse disturbance with amplitude -4 units, and  
c) saw tooth disturbance from Fig.4.a

From figures it is evident that fuzzy control with formal approach gives the best result, and that the result of the fuzzy control with linguistic approach is comparable, but little worse than the result of conventional feedforward control.

At the end it is important to emphasize that in this example it was possible to evaluate deterministic model of the process and apply conventional control technique, because the cause-effect data were real, but imprecise. The real worth of the fuzzy models and fuzzy feedforward control is in those situations when it is impossible to define any kind of deterministic models, for example for many systems in "soft" sciences.

#### CONCLUSION

Complex systems are difficult to control, because of inadequate knowledge of their behaviour. However, even a crude knowledge of the process behaviour is sufficient for the construction of fuzzy models of systems. On such a models fuzzy control may be build, so the principles of automatic control can be applied to the cases, where conventional control has not given desired quality.

Achievement of system invariance to disturbances acting upon it is one of the most important tasks in automatic control. In that cases the feedforward control is much more convenient than the feedback one. With the

methods proposed in this paper, the application of feedforward control may be widened also to the control problems of complex, ill-defined, nondeterministic systems, where only the fuzzy control shows the success.

The fuzzy models of systems suitable for practical application in feedforward control are developed by formal and linguistic approach. Identification procedures of fuzzy models are described and feedforward control algorithms which use results of simulation of system behaviour based on such fuzzy models are presented.

The proposed methods are illustrated by simulation of the control of simple industrial process. The comparison with conventional feedforward control obtained for the same data is also presented.

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