Governors and Early Stability Theory

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2. Maxwell-Routh
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1. Introduction

- Typical problems
  - Keeping constant speed of machines and instruments
  - Telescope drives, clocks Huygens Hookes late 1600
  - Wind mills, Lee 1745, Mead 1787
  - Steam engines: Watt's flyball governor
  - Water turbines Stodola
- Beginning of PID Control
- Integration of sensing control and actuation
- Problems
  - Gravity
  - Friction
  - Hunting
- Interaction theory and practice
- Concepts and ideas

Some Dates

- Mead late 1600
- Watt 1798
- Integrating action Perier, ≈ 1790
- Derivative action (Inertial action) Hicks 1840
- Hermite 1856
- Siemens 1866
- Maxwell 1868
- Vyshnegradskii 1876
- Routh 1877
- Lyapunov 1892
- Stodola 1894
- Hurwitz 1895

James Clarke Maxwell 1831-1879

- Born Glenlair South of Glasgow
- University of Edinburgh 1847
- Graduate Trinity College Cambridge 1854
- Professor Natural Philosophy Aberdeen 1856
- Professor Physics and Astronomy Kings College London 1860
- A Treatise on Electricity and Magnetism 1873
- Professor Experimental Physics Cavendish Laboratory 1871
- Broad interests
  - Color vision
  - Saturns rings
  - Control 1868 paper

Maxwells 1868 Paper

- Stability Concept
- Simple Mathematical Models
- Importance of integral action
- Linearization
- Stability is an algebraic problem
- Criteria for first, second and third order systems
- Posed stability problem in competition

Taxonomy - Stability Concept

"It will be seen that the motion of a machine with its governor consist in general of a uniform motion, combined with a disturbance which may be expressed as the sum of several component motions. These components may be of four different kind:

1. The disturbance may continually increase.
2. It may continually diminish.
3. It may be an oscillation of continually increasing amplitude.
4. It may be an oscillation of continually decreasing amplitude.

The first and third cases are evidently inconsistent with the stability of the motion: and the second and fourth alone are admissible in a good governor. *This condition is mathematically equivalent to the condition that all the possible roots, and all the possible parts of the impossible roots of a certain equation shall be negative.*"

Proportional and Integral Action

" Most governors depend on the centrifugal force of a piece connected with a shaft of the machine. When the velocity increases, this force increases , and either increases the pressure of the piece against a surface or moves the piece, and so acts on a break or a valve.

In one class of regulators of machinery, which we may call *moderators*, the resistance is increased by a quantity depending on the velocity.

But if the part acted on by centrifugal force, instead of acting directly on the machine, sets in motion a contrivance which continually increases the resistance as long as the velocity is above its normal value, and reverses its action when the velocity is below that value, the governor will bring the velocity to the same normal value whatever variation (within the working limits of the machine) be made in the driving-power or the resistance."

Mathematical Models

See "Blue Book" page 131-136

$$J\frac{d\omega}{dt} + D\omega = C\varphi - M_l$$
$$\frac{d^2\varphi}{dt^2} + b\frac{d\varphi}{dt} + a\varphi = -aK\omega$$

Hence

$$J\frac{d^3\omega}{dt^3} + (D+bJ)\frac{d^2\omega}{dt^2} + (aJ+bD)\frac{d\omega}{dt} + (aD+aKC)\omega = -\frac{d^2M_l}{dt^2} + b\frac{dM_l}{dt} + aM_l$$

Stability governed by the algebraic equation

$$a_0x^3 + a_1x^2 + a_2x + a_3 = 0$$

Maxwell states the conditions:

$$a_i > 0, \quad a_1a_2 > a_0a_3$$

$$(D+bJ)(aJ+bD) - J(aD+aKC) = abJ^2 + b^2DJ + bD^2 - aKCJ$$

Proofs

First order

$$x + a_1 = 0$$

Second order

$$x^2 + a_1 x + a_2 = 0$$

Third order

$$(x+a)(x^2+bx+c) = x^3+(a+b)x^2+(ab+c)x+ac$$

Hence

$$a_1 = a + b \qquad (1)$$
$$a_2 = ab + c \qquad (2)$$
$$a_3 = ac \qquad (3)$$

What does $a > 0$, $b > 0$ and $c > 0$ imply for a_1, a_2 and a_3?

- ab-plane, $c = 0$, $a_3 = 0$
- bc-plane, $a = 0$, $a_3 = 0$
- ac-plane, $b = 0$, $a_1 = a$, $a_2 = c$, $a_3 = a_1 a_2$

Edward John Routh 1831-1907

- Born in Quebeck
- Senior Wrangler Mathematical Tripos Cambridge (Maxwell second!)
- Maxwells problem 3rd order included in Routh's book
- Adams Prize competition: The criterion of Dynamic Stability. 1875-77. Maxwell Judge
- Routh won with the 1876 paper: A treatise on the stability of the given state of motion.
- Key ideas: Cauchy Index, Sturm chain
- Did not know of Hermite
- Good teacher

The Cauchy Index

The Cauchy index of a real rational function $R(x)$ between the limits a and b is the difference between the jumps of $R(x)$ from $-\infty$, to $+\infty$ and that of jumps in the opposite direction as x changes from a to b.

$$R(z) = \sum_{i=1}^{p} \frac{A_i}{z - p_i} + R_1(z)$$

where $R_1(x)$ has no real poles

$$I_a^b R(x) = \sum \operatorname{sign} A_i$$

If $f(x) = a_0(x-p_1)_1^n(x-p_2)_2^n \ldots (x-p_m)_m^n$ is a real polynomial where only the first p roots are real we have

$$\frac{f'(x)}{f(x)} = \sum_{i=1}^{p} \frac{n_i}{x - p_i} + R_1(x)$$

The index $I_a^b \frac{f'(x)}{f(x)}$ is the number of real distinct roots of $f(x)$ in the interval (a, b).

Sturm Sequences

A method to compute the Cauchy index.

Consider a sequence of real polynomials

$$f_1(x), f_2(x), \ldots, f_m(x)$$

This is a Sturm sequence if

1. For every value x if any polynomial $f_k(x)$ vanishes the adjacent polynomials $f_{k-1}(x)$ and $f_{k+1}(x)$ have values different from zero and of opposite sign, i.e.
$$f_{k-1}(x) f_{k+1}(x) < 0$$
2. The last function $f_m(x)$ does not vanish in the interval.

Let $V(x)$ the number of of sign in the sequence for a given x. Then

$$I_a^b \frac{f_2(x)}{f_1(x)} = V(a) - V(b)$$

Construct chain with Euclids algorithm:
$f_3(x) = -f_1(x) \bmod f_2(x)$

Routh's Approach

$$F(z) = a_0 z^n + a_1 z^{n-1} + \ldots + a_n$$
$$f(i\omega) = U(\omega) + iV(\omega)$$

Use Cauchy index to count encirclements. Look at

$$\theta = \arctan \frac{V(\omega)}{U(\omega)}$$

and

$$I \frac{V(\omega)}{U(\omega})$$

Routh array

$$\begin{matrix} a_0 & a_2 & a_4 & a_6 & \ldots \\ a_1 & a_3 & a_5 & a_7 \ldots & \end{matrix}$$

Vyshnegradskii 1876

More detailed analysis than Maxwell. Analysis of governor French Academy of Sciences 1876. Expanded version in Russian, German and French 1977-1879.

Idea of linearization

$$\frac{d^3u}{dt^3} + M\frac{d^2u}{dt^2} + N\frac{du}{dt} + \frac{KL}{J\omega_0}u = \frac{K(p-q)\rho}{J\omega_0}$$

Normalize

$$z^3 + az^2 + bz + 1 = 0$$

Distinguish different pole configurations

Stability diagram

Vyshnegradskii' Theses

1. Increase of the mass of balls harmful for stability
2. Decrease of friction in governor harmful for stability
3. Decrease of moment of inertia of flywheel is harmful for stability
4. Decrease of nonuniformity harmful for stability

Notice however that Vyshnegradskii did not discuss the importance of integral action.

Aurel Boleslaw Stodola 1859-1942

- Born in Slovakien
- Studies and industrial practice in Prague, Berlin, Paris, Zurich
- Professor Polytechnikum (now ETH) Zurich 1892
- Many invitations elsewhere
- Regulation of high pressure water turbines
- Modeling
  - Time constants
  - Dimensionless quantities (percentage deviations)
- 3rd order equation used Vyshnegradskii
- 7th order equation help from Hurwitz
- Application to Davos Spa Turbine plant
- Publication 1894
- Used derivative action

Adolf Hurwitz 1858-1919

$$f(x) = a_0x^n + a_1x^{n-1} + \ldots + a_n$$

Hurwitz obtained a condition in terms of subdeterminants of

$$\begin{matrix} a_1 & a_3 & a_5 & \ldots \\ a_0 & a_2 & a_4 & \ldots \\ 0 & a_0 & a_2 & \ldots \\ 0 & a_1 & a_3 & \ldots \\ \vdots & & & \end{matrix}$$

Charles Hermite 1822-1901

Interested in roots with positive imaginary part: Paper 1854:

On the number of roots of an algebraic equation between two limits.

Complex coefficients!

Key idea: Use of quadratic forms

$$f(x) = a_0x^n + a_1x^{n-1} + \ldots + a_n$$

$$\frac{f(x)f(y) - f(-x)f(-y)}{x+y} = \sum_{i,j=1}^{n} h_{ij}x^{n-i}y^{n-j}$$

$$\begin{pmatrix} x^{n-1} & \ldots & 1 \end{pmatrix} H \begin{pmatrix} y^{n-1}3 & \ldots & 1 \end{pmatrix}^T$$

Example n=3

$$H = \begin{pmatrix} a_0a_1 & 0 & a_0a_3 \\ 0 & a_1a_2 - a_0a_3 & 0 \\ a_0a_3 & 0 & a_1a_3 \end{pmatrix}$$

Differences in Impact of Maxwell-Routh and Stodola-Hurwitz

Bennet: Knowledge of the (stability) conditions did not, for many years spread beyond the scientific circle of Maxwell, Thomson, Airy Siemens and others. Ordinary engineers remained unaware of this theoretical work.

Maxwell had been concerned with special governors (instruments) and not with the typical engine governor.

Stodola's work was much more engineering oriented. Hurwitz criterion was first published by Stodola 1894 without proof in an engineering journal, see Bissell handout.

The idea of time constants and dimension-free variables very useful.

Results published in a book M. Tolle (1905) Die Regelung der Kraftmaschinen, Springer Berlin.

Alexandr Mihailovich Lyapunov 1857-1918

- St. Petersburg University, Maters thesis: On the stability of ellipsoidal forms of equilibrium of rotating fluids 1884
- Privat-Dozent Mechanics Kharkov 1885
- Doctoral dissertation 1892 The General Problem of the Stability of Motion. Opponent: Joukowski. Back to Kharkov.
- Academician Unive of St. Petersburg 1902, Chebychev's chair.
- To Odessa on doctors order in 1917, wife tuberculosis. Lyapunov in poor shape, bad eyesight.
- Family estate burned during Russian revolution including a great library built by his father and grand father. Suicide 1918.

Some Contributions

- Lyapunov exponents
- Stability concept
- Lyapunov's first method - Linearize
- Lyapunov's Second Method - Find asymptotic stability without using an explicit solution

$$\frac{dx}{dt} = f(x)$$

$$\frac{dV}{dt} = \frac{\partial V}{\partial x} f(x) < 0$$

Impact of Lyapunov's

- Nikolai Egorovich Zhukovskii (Joukowsky) Theory of regulation of the motion of machines, 1909. Equivalent of Tolle's book. Remained in circulation for a long time revised edition 1933 Kotelnikov and Smirnov.
- 1934 Automation and Remote Control became a special section of the Academy of Sciences. The journal Avtomatica i Telemechanica started.
- Kazan Aviation Institute
  - Chetaev 1946
  - Malkin 1952
- Andronovs Seminar Started 1944
  - Lurje
  - Aizerman

Impact of Lyapunov in the West

- Solomon Lefschetz translated Krylov and Bogolyubovs book 1943
- Cover-to-cover translation of Automatica I Telemechanica began 1957
- Lefschetz quote to USAF 1959:" A point of particular importance here is the development and application of Lyapunov's theory of stability and use of methods and techniques that have been almost completely ignored outside the USSR"
- Special efforts to propagate ideas by Lefschetz and LaSalle in US, Cartwright and Parks in England
- La Salle Princeton-RIAS-Brown University
- Impact of IFAC
- Kalman and Bertram ASME paper 1960

Kalman Bertram Paper

Kalman at RIAS, Bertram at IBM

Control System Analysis and Design via the "Second Method of Lyapunov". ASME Journal of Basic Engineering 1960, pp304-499.

The original work of Lyapunov in French is difficult to read because of the obsolescent mathematical terminology. An authoritative survey is given by Massera. The recent monograph by Hahn (in German) covers existing results and includes a few applications. There is now an English translation of the the well-known book by Malkin. This book, allegedly addressed to engineers, is primarily a detailed and rigorous mathematical treatment of classical problems. ...

Over the past 10 years, a great deal has been published about the "second method" in the journals Avtomatica i Telemechanika and Prikladnaya matematika i Mekhanika (now both available in English translation).

Connections Routh-Hurwitz-Lyapunov

Routh-Hurwitz matrix manipulations

Use Hermite's matrix as a Lyapunov function

$$\begin{pmatrix} a_1 & 0 & a_3 & \dots \\ 0 & -a_3 + a_1 a_2 & 0 & \dots \\ a_3 & 0 & a_5 - a_1 a_4 + a_2 a_3 & \dots \\ \vdots & & & \\ \dots & \dots & \dots & a_n a_{n-1} \end{pmatrix}$$

$$V = x^T H x$$

Quadratic forms

Ladder lattice representations

Quadratic loss functions

Interesting Continuations

Impact of the conditions $a_i > 0$

Nyquist

Discrete time systems Schur-Cohn-Jury

The Lurje problem

Aizerman's conjecture 1949

Popov

Yakobuvich

Small gain theorem

Passivity

Kharitonov

Industrial Impact - 1

- Industrial Governors
  - Porter abandoned the manufacturing of stone dressing machines and turned to the more profitable business of making governors
  - The Woodward Governor Patent 1890
  - Oldenburger worked for the Woodward company

Industrial Impact - 2

- Results of Maxwell and Routh were known in scientific circles they had very little impact on practical regulator design.
- Early example of the GAP between theory and practice
- Why?
  - Speculations
  - Did not answer the critical design issues
  - Friction and many other things neglected
  - Bad recommendations because of simplified models
- Routh work was first used for flight control
- German work in good contact with industry
- Tolle (1905) Regelung der Kraftmaschinen
- But, nobody was aware of Lyapunov!!

Conclusions

- It pays to know history and the literature!
- It pays to interact with other researchers
  - Neither Routh nor Hurwitz knew that Hermite had solved the problem
  - Imagine what could have happened if Routh-Hurwitz and Lyapunov had got together!
- Do not forget the customers and how the results are packaged and presented. Compare the impact of Maxwell and Stodola!
- How important are the results?
- What should we remember? - Personal bias: Principle of the argument!!

Home Work

- For everyone: What is the role of stability theory for automatic control? What should we teach?
- For everyone: Look at Maxwell's and Vyshnegradskii's papers. Try to imagine the impact of the papers on the practice of control.
- For the mechanically inclined: Explain inertial action (Stodola)
- For the mechanically inclined: Explain in detail how integral action is obtained in the centrifugal governor.
- For the mathematically inclined: Look at the proofs of the Routh-Hurwitz. What is a nice way to do this for teaching?

Reading Material

J. C. Maxwell. On Governors. Proc. Roy. Soc. 16 (1868) 270-283.

C.C. Bissell (1989) Stodola, Hurwitz and the genesis of the stability criterion. Int. J. on Control 50 (1989) pp 2313-2332. Contains translation of Vyshnegradskii's paper.

S. Bennett A history of Control Engineering 1800-1930 Peter Peregrinus, IEE 1979. Contains much material on governors.

R. Bellman and R. Kalaba Selected papers on mathematical trends in control theory. Dover 1964. Contains many original papers, e.g. Maxwell, Hurwitz, Nyquist etc.

F. R. Gantmacher The Theory of Matrices Vol. 2. Chelsea 1959. Contains proofs of all the theorems and a good discussion of relations and connections.

K. J Åstr—"om Introduction to Stochastic Control Theory. Academic Press 1970. Simple proofs and connections with squared signals.