The Feedback Amplifier

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Introduction

- Driving force: Emerging telecommunications
- Alexander Graham Bell 1847-1922
- Patent 1876
- No patent in Sweden!
- AT&T formed a Research Laboratory in 1907 as part of a strategy to control all American Telecommunications: It is necessary to control the rate and direction of technology by obtaining, or preventing others to obtain, key patents.
- The repeater problem
- Black's invention
- Bode
- Nyquist

The Repeater Problem

- The electromechanical repeater
- 6mm wire 280 kg/km
- John J Carter 1909: Whoever can supply and control the telephone repeater will exert a dominating influence ...
- 1911 East coast to Denver
- 1914 First transcontinental New York San Francisco
- 1915 Improved transcontinental three vacuum tube repeaters, two repeaters added in 1916 and two more in 1918.

System	*Date*	*Ch pair*	*Loss db 3000mi*	*Repeat 3000mi*
1st TC	1914	1	60	3-6
2nd TC	1923	1-4	150-400	6-20
Open W	1938	16	1000	40
Cable	1936	12	12.000	200
Coaxial	1941	480	30.000	600

Bode, The History of an Idea

Most of you with hi-fi systems are no doubt proud of your audio amplifieres, but I doubt whether many of you would care to listen to the sound after the signal had gone in succession through several dozen or several hundred even of your fine amplifiers. There is a 'tyranny of numbers, as my reliability friends say, which makes it necessary for the individual components of the system to become qualitatively better as the system as a whole becomes quantitatively more ambitious.

Bode, cont

The causes of distortion were of various sorts. They included power supply noises, variations in gain and so on. The dominant problem, however, was the intermodulation due to the slight nonlinearity in the characteristics of the last tube. Various efforts were made to improve this situation, by the selection of tubels, by careful biasing, by the use of matched tubes in push-pull to provide compensating characteristics, and so on. Until Black's invention, however, nothing made a radical improvement of the situation.

Harold S. Black

- Worchester Polytechnic Institute 1921
- Joined Western Electric that later became part of Bell Telephone Laboratories
- Remained with Bell until 1963
- Principal Research Scientist with General Precision Corporation
- The feedback amplifier
- Theory and application of pulse-code modulation
- IEEE Lamme Medal 1957
- 62 US Patents
- 271 patents in 32 countries
- 1953 Modulation Theory

Black on Black - 1

The telephone industry was an exciting place for a young engineer in 1921. Only a few years had passed since Lee de Forest's 1906 audion tube had been made into the practical high-vacuum device needed for long-distance telephone lines. In 1915, the 68-year-old Alexander Graham Bell had placed the first official transcontinental call to Thomas Watson, his famous assistant 40 years earlier, and in 1914, a young Edwin Howard Armstrong had sat through bitterly cold January night with David Sarnoff in an American Marconi wireless shack testing Armstrong's new "regenerative" receiver, which utilized positive feedback.

Black on Black - 2

No one knew how to make amplifiers linear or stable enough in those days, and consequently they were subject to an intolerable amount of distortion. The problem lay with the unwanted frequencies generated by the vacuum tubes, particularly the second-order harmonics and other products that predominated. Theoretically, this type of amplifier should have suppressed all second-order harmonics and products, but in actuality they were only reduced to 10–18 percent. The first thing I did ... , I plotted one curve showing how linearity would vary with the number of channels and another curve showing how it would be affected by adding more push-pull amplifiers in the string. Here, I made a significant error.

Black on Black - 3

The next day at the laboratory I dropped in on Ralph Hartley, who was in the nearby Research Department. A Rhodes scholar whom I liked and respected very much (he would later invent magnetic parametric amplifiers and logarithmic information measure), my problem was right in his field, for he had worked on third-harmonic modulators. I described my results to Hartley and on the following day he sent me a note pointing out my error ... . This meant that if I had a string of 1000 amplifiers the cumulative voltage distortion would amount to 60 dB. It was bad enough to have a lot of channels going through one amplifier, but to pile on distortion of this magnitude made it clear that I faced many serious problems in building strings of multichannel amplifiers.

Calculating the Distorsion

Define distorsion as

$$d = \frac{\max f'(x) - \min f'(x)}{\max f'(x) + \min f'(x)}$$

Compositions of functions

$$g = f \circ f \ldots \circ f$$

Hence $g' = (f')^n$

Example

$$f(x) = \frac{x + ax^2}{1 + a}$$

Assume $a = 0.01$ then $d = 0.01$, but

$$g' = n\left(\frac{x + ax^2}{1 + a}\right)^{n-1} \frac{1 + 2ax}{1 + a}$$

With $n = 100$ we get $d = 0.76$.

An encounter with Steinmetz

This might have been the end of it except that, on March 16, 1923, I was fortunate enough to attend a lecture by the famous scientist and engineer, Charles Proteus Steinmetz. The meeting was held at the AIEE auditorium in New York City, and I remember arriving early in order to get a seat in the front row. Soon every seat was taken. Dr. Steinmetz was 20 minutes late, but he was given a standing ovation as he walked down the center aisle dressed in blue overalls and a blue shirt with short sleeves and smoking the largest cigar I had ever seen. It was an inspiring lecture. I no longer remember the subject, but I do remember the clarity and logic of his presentation and how quickly and directly he reached the final conclusion of his talk.

Inspiration of the Lackawanny Ferry

Then came the morning of Tuesday, August 2, 1927, when the concept of the negative feedback amplifier came to me in a flash while I was crossing the Hudson River on the Lackawanna Ferry, on my way to work. For more than 50 years I have pondered how and why the idea came, and I can't say any more today than I could that morning. All I know is that after several years of hard work on the problem, I suddenly realized that if I fed the amplifier output back to the input, in reverse phase, and kept the device from oscillating (singing, as we called it then), I would have exactly what I wanted: a means of canceling out the distortion in the output. I opened my morning newspaper and on a page of The New York Times I sketched a simple canonical diagram ... when I reached the laboratory at 463 West Street, it was witnessed, understood, and signed by the late Earl C. Blessing.

Black's Original

The Idea of Feedback

However, by building an amplifier whose gain is deliberately made, say 40 decibels higher than necessary and then feeding the output back on the input in such a way as to throw away the excess gain, it had been found possible to effect extraordinary improvement in constancy of amplification and freedom from non-linearity.

Stabilized feedback processes other advantages including reduced delay and delay distortion, reduced noise disturbance from the power suply circuits and various other features best appreciated by practical designers of amplifiers.

$$\frac{Y}{U} = \frac{\mu}{1 - \beta\mu} = -\frac{1}{\beta} \, \frac{\mu}{\mu - 1/\beta}$$

Gain is the hard-currency that can be traded for many other qualities!dvips -p 14 -l 14 Le

The operational amplifier!

Inspiration of the Lackawanny Ferry

I envisioned this circuit as leading to extremely linear amplifiers (40 to 50 dB of negative feedback), but an important question is: How did I know I could avoid self-oscillations over very wide frequency bands when many people doubted such circuits would be stable? My confidence stemmed from work that I had done two years earlier ... In the course of this work, I had computed and measured transfer factors around feedback loops and discovered that for self-oscillations to occur the loop transfer factor must be real, positive, and greater than unity at some frequency. Consequently, I know that in order to avoid self-oscillation in a feedback amplifier it would be sufficient that at no frequency from zero to infinity should $\mu\beta$ be real, positive, and greater than unity. Accordingly, I immediately proceeded with the careful design and development of a working model ...

Progress in the Lab

On December 29, 1927, using typical input signals covering a frequency band extending from 4 to 45 kHz, a reduction of distortion of 100 000 to 1 (50 dB) was realized in a single amplifier – which was more than sufficient to do the job I had undertaken six years earlier. These final results were immediately transmitted to Harry A. Burgess, patent attorney at Bell Laboratories.

Nine years in the Patent Office

Although the invention had been submitted to the U.S. Patent Office on August 8, 1928, more than nine years would elapse before the patent was issued on December 21, 1937 (No. 2 102 671). One reason for the delay was that the concept was so contrary to established beliefs that the Patent Office initially did not believe it would work. The Office cited technical papers, for example, that maintained the output could not be connected back to the input unless the loop gain was less than one, whereas mine was between 40 and 50 dB. In England, our patent application was treated in the same manner as one for a perpetual-motion machine. Burgess was eventually able to overcome all these objections by submitting evidence that 70 amplifiers were working successfully ...

Nine Years, cont

The second reason for the nine-year wait was that numerous Patent Office objections to the length and arguments about the claims ...

I wrote most of the body of the patent, supplied all the illustrations, and suggested most of the claims. I foresaw that the mathematical understanding developed in connection with feedback electronic amplifiers could be carried over and applied by analogy to he synthesis and analysis of other kinds of amplifiers, to all kinds of control systems – mechanical, acoustical, chemical, hydraulic, or whatever. The patent clearly applies to large complex industrial and military control systems, implying the capability to exercise specific control of a single variable or an entire system.

To achieve these ends, the claims were written very broadly and I worked tenaciously with the U.S. Patent Office to keep the broad applicability as granted, even though this added to the delay.

Mervin Kelly on Black IEEE Lamme Medal 1957

Although may of Harold's inventions have made great impact, that of the negative feedback amplifier is indeed the most outstanding. It easily ranks coordinate with De Forest's invention of the audion as one of the two inventions of broadest scope and significance in electronics and communications of the past 50 years....it is no exaggeration to say that without Black's invention, the present long-distance telephone and television networks which cover our entire country and the transoceanic telephone cables would not exist. The application of Black's principle of negative feedback has not been limited to telecommunications. Many of the industrial and military amplifiers would not be possible except for its use.

Mervyn Kelly, cont

Many of the industrial and military amplifiers would not be possible except for its use. Many new weapons, such as radio detection of bombing and radar control of missiles, are dependent on negative feedback for their success. The development of servomechanisms theory and its application are extensions of Black's principle of feedback and are generally recognized as such. Thus, the entire explosive extension of the area of control, both electrical and mechanical, grew out of an understanding of the feedback principle. This principle also sheds light in psychology and physiology on the nature of the mechanisms that control the operation of animals, including humans, that is, on how the brain and senses operate.

Nyquist and His Contributions

- How to view electronic circuits
- Frequency response
- The feedback amplifier and singing
- Stability concepts
- Stability theory
- Why was this so important?
- When the Nyquist theorem came to ASEA
- Sampling

Harry Nyquist

- Born 1889 i Tomthult i Nilsby
- Finished school Nilsby 1902
- "Stalldräng", Deje sulphate mill
- "Dräng" in Minnesota
- Teacher seminar Austin 1908-09
- Folkskollärare Sout Dakota 1909-10
- Teacher seminar second year 1910-11
- High school teacher Belgrade Minn 1911-1912
- University of Grand Forks North Dakota MS &BS 1912-15
- Yale University 1915-17
- Western Electric 1917-1919
- Bell Labs New York 1919-1945
- MIT 1940-45

Motivation

- Nyquist was a problem solver
- Stron drive to reduce a problems to its simplest form
- Understanding conditions for singing
- The problem of conditional stability
- How can the system be stable when the gain is higher than one at frequencies where the phase lag is 180°.
- Critical point was at $+1$! Bode moved it to -1!

Regeneration Theory BSTJ 1932

The circuit will be said to be stable when an impressed small disturbance, which itself dies out, results in a response which dies out. It will be said to be unstable when such a disturbance results in a response which goes on indefinitely, either staying at a relatively small value or increasing.

Rule: Plot plus and minus the imaginary part of $AJ(i\omega)$ against the real part for all frequencies from 0 to ω. If the point $1 + i0$ lies completely outside this curve the system is stable; if not it is unstable (p. 136).

Conditional Stability

The circuit will be said to be stable whe For low values of A (the amplifier gain) the system is in a stable condition. Then as the gain increased gradually, the system becomes unstable. Then as the gain is increased gradually still further, the system again becomes stable. As the gain is still further increased the system may again become unstable (p. 137).

Impact of the Nyquist Theorem at ASEA

Bode's Contributions

Network Analysis and Feedback Amplifier Design

- Gain Phase Relations

$$B = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{dA}{du} \log coth \frac{|u|}{2} du$$

$$A_c - A_0 = \frac{2}{\pi} \int_{-\infty}^{\infty} B du$$

- Bode's integral

$$\int_{-\infty}^{\infty} \log S(i\omega) d\omega = 0$$

  The waterbed effect!

- Bode curves
- Systematic design procedure
- Bodes ideal Cut-off Characteristics

Bode's Relations

While no unique relation between attenuation and phase can be stated for a general circuit, a unique relation does exist between any given loss characteristic and the *minimum* phase shift which must be associated with it. (p. 424)

$$B = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{dA}{du} \log coth \frac{|u|}{2} du$$

The significance of the phase area relation for feedback amplifier design can be understood by supposing that the practical transmission range of the amplifier extends from zero to some given finite frequency. The quantity $a_0 - A_\infty$ can be identified with the change in gain around the feedback loop required to secure cut-off. Associated with it must be a certain definite phase area.

$$A_c - A_0 = \frac{2}{\pi} \int_{-\infty}^{\infty} B du$$

Bode's Integral

- Recall definition of sensitivity
- $S = \frac{Y_{cl}}{Y_{ol}} = \frac{1}{1+L} = \frac{\partial T}{\partial \theta}$
- There are constraints on the sensitivity function
- $\int_0^\infty \log |S(i\omega)| d\omega = \pi \sum \mathrm{Re}\ p_i$
- The water bed effect
- Stein's Bode Lecture

The Role of Bell Labs

- Science and technical leadership
- Black
- Nyquist
- Bode
- The transistor
- McMillan
- MacColl Fundamentals of servomechanisms, 1945
- Many others!

Bode's Ideal Cut-off Characteristics

Conclusions

- Driving force
- A very fertile development
- Business structure
- Bell Labs versus 10 instrument company
- The principle of feedback
- A theoretical framework
  - Frequency response
  - Complex variables
- Stability theory
- Design methods
- Fundamental limitations